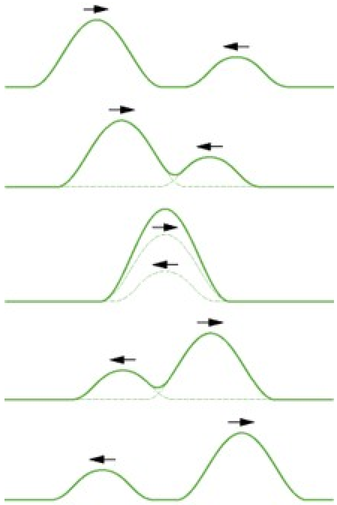
Lecture 18 Superposition and Standing Waves



18.1 Superposition and Interferences

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

Overlapping waves algebraically add to produce a resultant wave (or net wave).

Overlapping waves do not in any way alter the travel of each other.

Superposition of Sinusoidal Waves

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

$$y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

$$y'(x,t) = y_1 + y_2 = 2y_m \sin(kx - \omega t + \frac{\phi}{2})\cos(\frac{\phi}{2})$$

If two sinusoidal waves of the same amplitude and wavelength travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

Phase Differences and Resulting Interference Types

Phase difference			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$2\pi/3$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$4/3\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

Example:

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

(a) What is the amplitude y'_m of the resultant wave due to the interference of these two waves, and what type of interference occurs?

(b) What phase difference, in radius and wavelength, will give the resultant wave an amplitude of 4.9 mm?

$$y' = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) = 2y_m \cos(\frac{\phi}{2}) \sin(kx - \omega t + \frac{\phi}{2})$$

$$y'_m = 2y_m \cos(\frac{\phi}{2}) = 2 \cdot 9.8 \cdot \cos(50^o) = 12.6mm$$

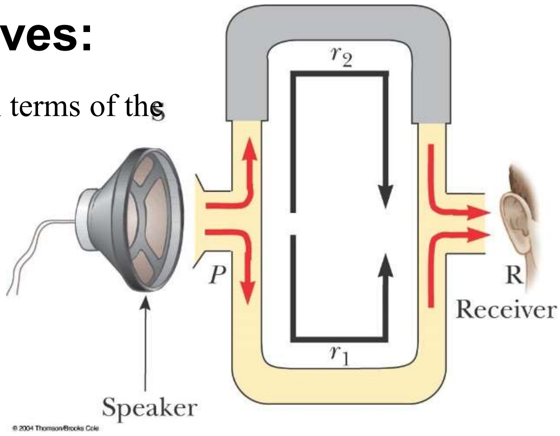
intermediate

$$y'_m = 4.9 = 2y_m \cos(\frac{\phi}{2}) = 2 \cdot 9.8 \cdot \cos(\frac{\phi}{2}), \quad \phi = 151^o = 2.636rad$$

$$\frac{2\pi}{\lambda} x = 2.636, \quad x = 0.42 \cdot \lambda$$

Interference of Sound Waves:

It is often useful to express path difference in terms of the phase angle ϕ between the two waves.

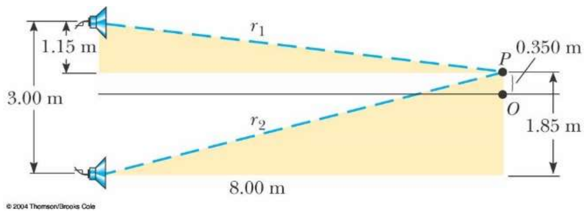


$$\frac{\Delta r}{\lambda} = \frac{\phi}{2\pi}$$

$$\Delta r = 2n\frac{\lambda}{2} \quad \text{for constructive interference}$$

$$\Delta r = (2n+1)\frac{\lambda}{2} \quad \text{for destructive interference}$$

Example: Two identical speakers placed 3.00 m apart are driven by the same oscillator. A listener is originally at point O, located 8.00 m from the center of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 m from O, and she experiences the first minimum in sound intensity. What is the frequency of the oscillator?



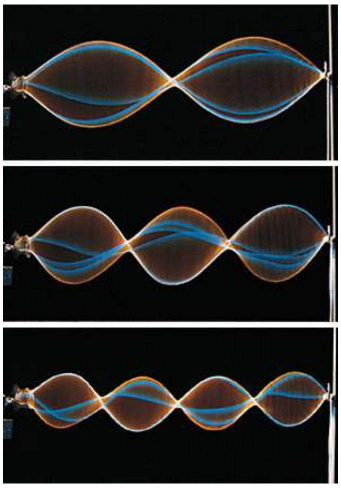
$$\Delta r = r_2 - r_1 = \sqrt{1.85^2 + 8^2} - \sqrt{1.15^2 + 8^2} = 0.13 \rightarrow \frac{\lambda}{2} = 0.13$$

$$\rightarrow f = v / \lambda = 343 / 0.26 = 1.3 \ \text{(kHz)}$$

18.2 Standing Waves

$y_1 = y_m \sin(kx - \omega t)$, $y_2 = y_m \sin(kx + \omega t)$

$y' = y_1 + y_2 = 2y_m \sin(kx)\cos(\omega t)$



$2y_m \sin(kx) = 0$, when $kx = n\pi$, $x = \frac{n}{2}\lambda$ are the position of nodes

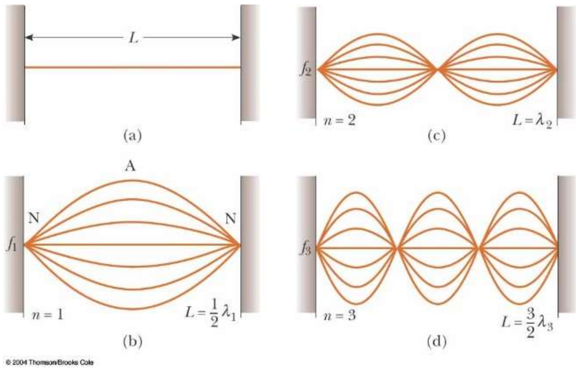
$2y_m \sin(kx) = \max imum$, when $kx = (n + \frac{1}{2})\pi$, $x = (n + \frac{1}{2})\frac{\lambda}{2}$ are the position of antinodes

18.3 Standing Waves in Strings Fixed at Both Ends

boundary condition: $\sin(kx = 0) = 0$, $\sin(kL) = 0$

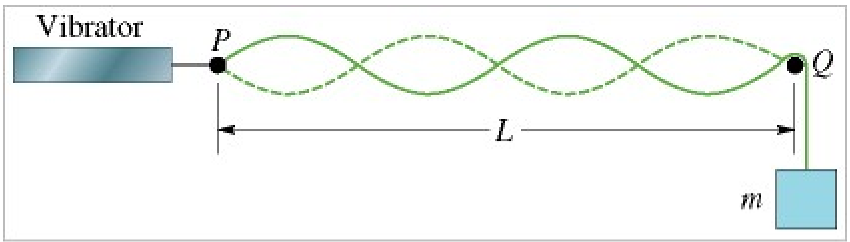
$kL = n\pi$, $\lambda = \frac{2L}{n}$

$f = \frac{v}{\lambda} = n\frac{v}{2L}$



Example:

A string, tied to a sinusoidal vibrator at P and running over a support at Q, is stretched by a block of mass m. The separation L between P and Q is 1.2 m, the linear density of the string is 1.6 g/m, and the frequency f of the vibrator is fixed at 120 Hz. The amplitude of the motion at P is small enough for that point to be considered a node. A node also exists at Q.



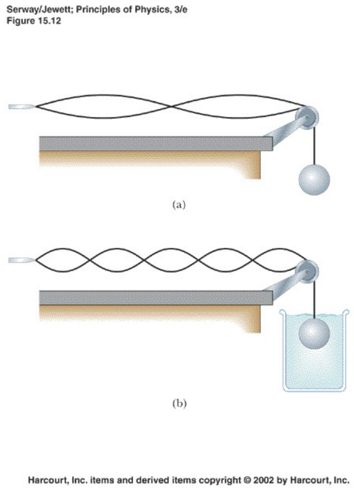
(a) What mass m allows the vibrator to set up the fourth harmonic on the string?

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{mg}{0.0016}},\quad \lambda=\frac{2L}{n}=\frac{2\cdot 1.2}{4}=0.6,\quad v=f\lambda=120\cdot 0.6=72$$

$$m\cdot 9.8=72^2\cdot 0.0016,\quad m=0.846kg$$

Example: Changing string vibration with water

One end of a horizontal string is attached to a vibrating blade, and the other end passes over a pulley. A sphere of mass 2.00 kg hangs on the end of the string. The string is vibrating in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. After this is done, the string vibrates in its fifth harmonic. What is the radius of the sphere?



$$T_1=mg=2\cdot 9.8=19.6N,\quad v=\sqrt{\frac{T}{\mu}},\quad f=\frac{v_1}{\lambda_1}=\frac{v_2}{\lambda_2},$$

$$\frac{\lambda_2}{\lambda_1}=\sqrt{\frac{T_2}{T_1}},\quad \frac{L/2.5}{L/1}=\sqrt{\frac{T_2}{19.6}}$$

$$T_2=3.136=T_1-\rho g\frac{4\pi}{3}R^3=19.6-1000\cdot 9.8\frac{4\pi}{3}R^3$$

$$R=0.0737m$$

Example:

A middle C string on a piano has a fundamental frequency of 262 Hz, and the A note has a fundamental frequency of 440 Hz. (a) Calculate the frequency of the next two harmonics of the C string. (b) If the strings for A and C notes are assumed to have the same mass per unit length and the same length, determine the ratio of tensions in the two string. (c) In a real piano, the assumption we made in part (b) is only partial true. The string densities are equal, but the A string is 64% as long as the C string. What is the ratio of their tensions?

(a) C string: $f_1=262Hz$, $f_2=2\cdot 262=524Hz$, $f_3=3\cdot 262=786Hz$

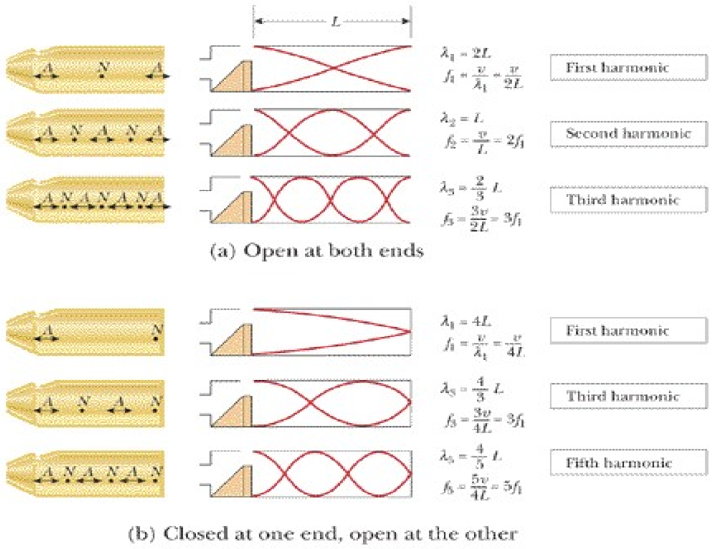
(b) $f=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$, $\frac{f_C}{f_A}=\sqrt{\frac{T_C}{\mu}}\sqrt{\frac{\mu}{T_A}}$, $\frac{262}{440}=\sqrt{\frac{T_C}{T_A}}$, $\frac{T_C}{T_A}=0.355$

(c) $\frac{f_C}{f_A} = \frac{L_A}{L_C}\sqrt{\frac{T_C}{T_A}} = 0.64\sqrt{\frac{T_C}{T_A}}$, $\frac{T_C}{T_A} = 0.866$

18.4 Resonance

18.5 Standing waves in air columns

Serway/Jewett; Principles of Physics, 3/e
Figure14.9



Example: Wind in a Culvert

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows across its open ends. (a) Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. Take v=343 m/s as the speed of sound in air.

$\lambda_1 = L/(1/2)$ → $f_1 = v/\lambda_1$

$\lambda_2 = L/(1)$ → $f_2 = v/\lambda_2 = 2f_1$

$\lambda_1 = L/(3/2)$ → $f_3 = v/\lambda_3 = 3f_1$

(b) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

$\lambda_1 = L/(1/4)$

$\lambda_2 = L/(3/4)$ → $f_2 = 3f_1$

18.6 Standing Waves in Rods and Membranes

18.7 Beats: Interference in Time

interference effect that results from the superposition of two waves with slightly different frequencies

$y_1 = A\cos(kx - \omega_1 t)$, $y_2 = A\cos(kx - \omega_2 t)$

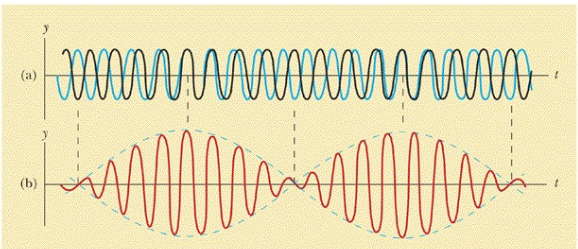
$y' = y_1 + y_2 = A\cos(kx - \omega_1 t) + A\cos(kx - \omega_2 t)$

$$y' = 2A\cos(kx - \frac{\omega_1 + \omega_2}{2}t)\cos(\frac{\omega_1 - \omega_2}{2}t) \approx 2A\cos(kx - \omega_1 t)\cos(\frac{\omega_1 - \omega_2}{2}t)$$

$$y_0 = 2A\cos(\frac{\omega_1 - \omega_2}{2}t)$$

the beat frequency = $\frac{1}{\pi}\frac{\omega_1 - \omega_2}{2} = f_1 - f_2$

Serway/Jewett; Principles of Physics, 3/e
Figure 14.11



18.8 Nonsinusoidal Wave Patterns

Fourier's theorem

$y(x)$ is a wave on the string satisfy $y(0) = y(L) = 0$

→ $y(x,t) = (a\cos(kx) + b\sin(kx))\sin(\omega t)$ → $y(x) = a\cos(kx) + b\sin(kx)$

→ $kL = n\pi$ → $y_n(x) = b_n \sin\left(\frac{n\pi}{L}x\right)$ → $y(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$

$f(x) = 10$, 0<x<L

$f(x) = 0$, x<0 or x>L

Example: $f(x)=\begin{cases}0, otherwise \\ 1, 0 \le x \le a\end{cases}$ can be synthesized by a harmonic series of sinusoidal waves: $\sum b_n \sin\left(\frac{n\pi x}{a}\right)$.

$$f(x)=\sum b_n \sin\left(\frac{n\pi x}{a}\right) \rightarrow \int_0^a \sin\left(\frac{n\pi x}{a}\right)(1)dx = b_n \int_0^a \sin^2\left(\frac{n\pi x}{a}\right)dx$$

$$\int_0^a \sin\left(\frac{n\pi x}{a}\right)(1)dx = b_n \frac{a}{2} \rightarrow b_n = \frac{2}{n\pi}[1-(-1)^n]$$

$$\rightarrow f(x)=\begin{cases}0, otherwise \\ 1, 0 \le x \le a\end{cases} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left(1-(-1)^n\right)}{n}\sin\left(\frac{n\pi x}{a}\right)$$