Chapter 27 Current and Resistance

We connect the wire filament in the light bulb across a potential difference causing the electric charge to flow through the wire, which is similar to water pressure resulting in the water flow through the horse.

Steady state: charge no longer continues to accumulate at points along the circuit and the current is steady

27.1 Electric Current

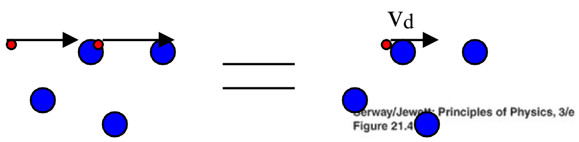
Current: The rate of flow of electric charge through a cross-sectional area
If ΔQ is the charge that flows through the cross section area A in time Δt, the current I is $I = \dfrac{\Delta Q}{\Delta t}$

Unit: ampere (A), 1 A = 1 C / s

instantaneous current $I = \dfrac{dQ}{dt}$

Microscopic Model of Current

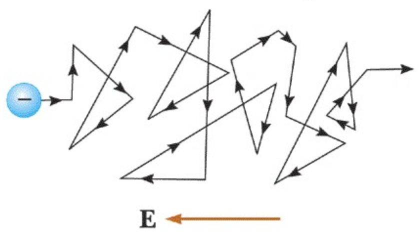
Drift speed:



n: the number density of charge carriers
q: the charge
In a time Δt, the number of particles in the volume $A(v_d \Delta t)$ is $nA(v_d \Delta t)$ and the total charge is $qnA(v_d \Delta t)$.

The current is $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{qnA(v_d \Delta t)}{\Delta t} = qnAv_d$

$I = \dfrac{dQ}{dt} = nqv_D A$, 1 Ampere = 1Coulomb / sec



Example: Drift speed in a copper wire

A copper wire of cross-section area 3.00 x 10^{-6} m^2 carries a current of 10.0 A. Find the drift speed of the electron in this wire. The density of copper is 8.95 g/cm^3.

$$v_D = \frac{10.0}{3.00\cdot 10^{-6}} \frac{1}{\frac{8.95}{63.5}\cdot 10^6 \cdot 6.02\cdot 10^{23}\cdot 1.602\cdot 10^{-19}} = 2.45\cdot 10^{-4}\, m/s$$

Example: The Drift Speed

A typical wire is made of copper and has a radius 0.815 mm. Calculate the drift speed of electrons in such a wire carrying a current 1 A, assuming one free electron per atom.

$$n = \frac{\rho_M N_A}{M} = \frac{(8.93g/cm^3)(6.02\times 10^{23})}{63.5g} = 8.47\times 10^{28}\, atom/m^3$$

$$v_d = \frac{I}{enA} = \frac{1}{(1.602\times 10^{-19}C)(8.47\times 10^{28})\pi(8.15\times 10^{-4})^2} = 3.54\times 10^{-2}\, mm/s$$

$$v_F = ?$$

Example: The Number Density

In a certain particle accelerator, a current of 0.5 mA is carried by a 5-MeV proton beam that has a radius of 1.5 mm. (a) Find the number density of protons in the beam.

$$K = 5MeV = (5\times 10^6)(1.602\times 10^{-19}) = \frac{1}{2}m_p v^2 = \frac{1}{2}(1.6\times 10^{-27})v^2$$

$$v = 3.1\times 10^7 \text{ m/s}$$

$$n = \frac{I}{qvA} = \frac{5\times 10^{-4}}{(1.602\times 10^{-19})(3.1\times 10^7)\pi(1.5\times 10^{-3})^2} = 1.43\times 10^{13}/m^3$$

27.2 Resistance

$$I = \frac{dQ}{dt} = nqv_D A,\ 1 \text{ Ampere} = 1\text{Coulomb / sec}$$

current density: $J = \frac{I}{A} = nqv_D$, v_D is drift velocity

The Ohm's law is $J = \sigma E$, where σ is conductivity and $\rho = 1/\sigma$ is resistivity.

The voltage difference across a distance *l* will be $\Delta V = El$.

The current density will be related to voltage as $J = \sigma \frac{\Delta V}{l}$.

The current can then be described as $I = AJ = \frac{A\sigma}{l}\Delta V$.

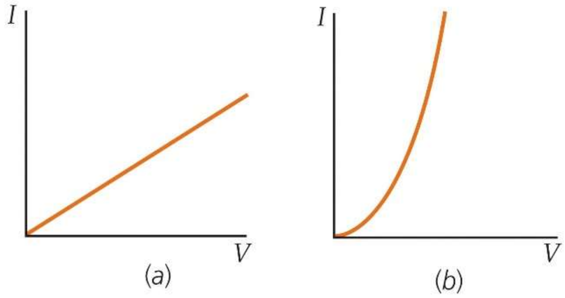
Thus, we find a simple relation between the current and voltage drop as

$$\Delta V = I\frac{l}{A\sigma} = IR. \rightarrow \quad R = \frac{1}{\sigma}\frac{l}{A} = \rho\frac{l}{A}$$

Assuming that the electric field is uniform,

$$\Delta V = V_b - V_a = E\Delta L$$

$$R = \frac{\Delta V}{I}$$ Unit: 1 Ω = 1 V / A



For ohmic materials: $V = IR$

The resistance & the resistivity: $R = \rho\frac{L}{A}$

Example: A Nichrome wire ($\rho = 10^{-6}$ Ωm) has a radius of 0.65 mm. What length of wire is needed to obtain a resistance of 2.0 Ω?

$$L = \frac{RA}{\rho}$$

What are metals and semiconductors?

What is a platinum resistance thermometer?

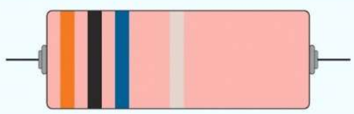
TABLE 25-1

Resistivities and Temperature Coefficients

Material	Resistivity ρ at 20°C, Ω·m	Temperature Coefficient α at 20°C, K^{-1}
Silver	1.6×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Aluminum	2.8×10^{-8}	3.9×10^{-3}
Tungsten	5.5×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Lead	22×10^{-8}	4.3×10^{-3}
Mercury	96×10^{-8}	0.9×10^{-3}
Nichrome	100×10^{-8}	0.4×10^{-3}
Carbon	3500×10^{-8}	-0.5×10^{-3}
Germanium	0.45	-4.8×10^{-2}
Silicon	640	-7.5×10^{-2}
Wood	$10^{8} - 10^{14}$	
Glass	$10^{10} - 10^{14}$	
Hard rubber	$10^{13} - 10^{16}$	
Amber	5×10^{14}	
Sulfur	1×10^{15}	

TABLE 25-3

The Color Code for Resistors and Other Devices



Colors	Numeral
Black	= 0
Brown	= 1
Red	= 2
Orange	= 3
Yellow	= 4
Green	= 5
Blue	= 6
Violet	= 7
Gray	= 8
White	= 9

Tolerance	
Brown	= 1 %
Red	= 2 %
Gold	= 5 %
Silver	= 10 %
None	= 20 %

The color bands are read starting with the band closest to the end of the resistor. The first two bands represent an integer between 1 and 99. The third band represents the number of zeros that follow. For the resistor shown, the colors of the first three bands are, respectively, orange, black, and blue. Thus, the number is 30,000,000 and the resistance is 30 MΩ. The fourth band is the tolerance band. If the fourth band is silver, as shown here, the tolerance is 10 percent. Ten percent of 30 is 3, so the resistance is (30 ± 3) MΩ.

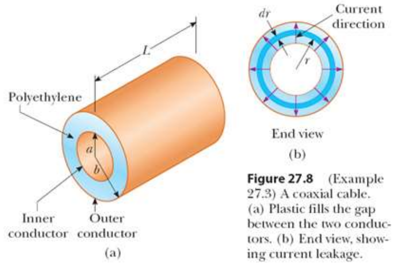
Example: The Electric Field That Drives The Current

A 14-gauge copper means its wire diameter, D = 1.628 mm.

Find the electric field strength E in the 14-gauge copper wire when the wire is carrying a current of 1.3 A.

$$E = V/l = IR/l = I\frac{R}{l} = I\frac{\rho}{A} = (1.3)\frac{1.7\times10^{-8}\Omega m}{\pi\left(8.14\times10^{-4}\right)^2} = 1.06\times10^{-2} V/m$$

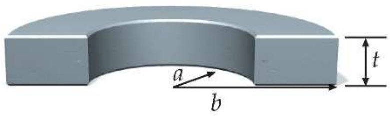
Example: Coaxial cables are used for television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in the right figure. Current leakage through the plastic, in the radial direction, is unwanted. The radius of the inner conductor is a = 0.500 cm, the radius of the outer conductor is b = 1.75 cm, and the length is L = 15.0 cm. The resistivity of the plastic is ρ = 1.0 X 10^{13} Ωm. Calculate the resistance of the plastic between the two conductors.



$$R = \frac{\rho l}{A} \rightarrow dR = \frac{\rho}{A}dl \rightarrow dR = \rho\frac{dr}{2\pi rL}$$

$$R = \int_a^b \frac{\rho}{2\pi L}\frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

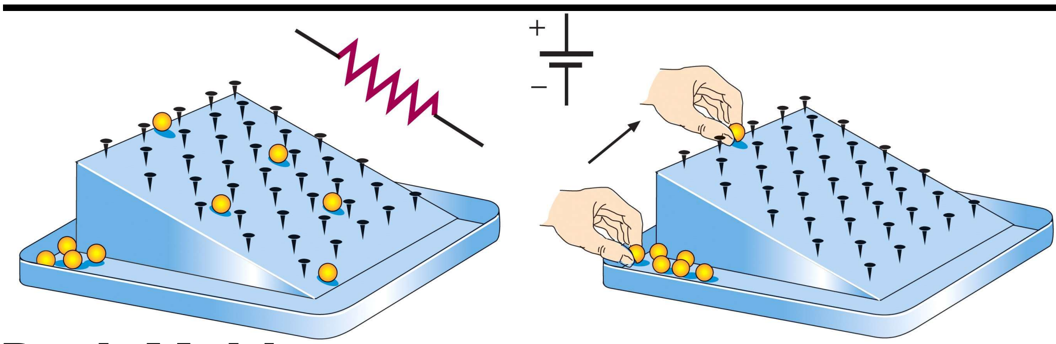
Example: The resistance of a semi-circular disc.



$$R = \rho\frac{l}{A},\quad G = \sigma\frac{A}{l} \;\text{-->}\; d\left(\frac{1}{R}\right) = \frac{1}{\rho}\frac{tdr}{\pi r}$$

$$\frac{1}{R} = \frac{t}{\pi\rho}\int_a^b\frac{dr}{r} \;\text{-->}\; R = \frac{\rho\pi}{t\ln\frac{b}{a}}$$

<u>27.3 A Model for Electrical Conduction</u>



Drude Model:

The electric field will drive free electrons move.

$m\vec{a} = \vec{F} = q\vec{E}$

It will accelerate the electrons' velocity.

$$\vec{v}_f = \vec{v}_i + \vec{a}t = \frac{q\vec{E}}{m}t$$

The drift velocity could be related to the acceleration and an average time interval τ between successive collisions.

$$\vec{v}_d = \frac{q\vec{E}}{m}\tau$$

$$J = nqv_d = nq\frac{qE\tau}{m} = \frac{nq^2\tau}{m}E = \sigma E \rightarrow \sigma = \frac{nq^2\tau}{m}, \quad \rho = \frac{1}{\sigma} = \frac{m}{nq^2\tau}$$

27.4 Resistance and Temperature

What are the RT behaviors of a metal and a semiconductor (or insulator)?

The temperature coefficient of resistivity: $\frac{1}{\rho}\frac{d\rho}{dT}|_{T=20^o C} = \frac{1}{\rho_{20^o C}}\frac{\rho - \rho_{20^o C}}{T - 20^o C}$

Change in resistivity with temperature of metals: $\rho = \rho_0[1 + \alpha(T - T_0)]$, $R = R_0[1 + \alpha(T - T_0)]$

27.5 Superconductors

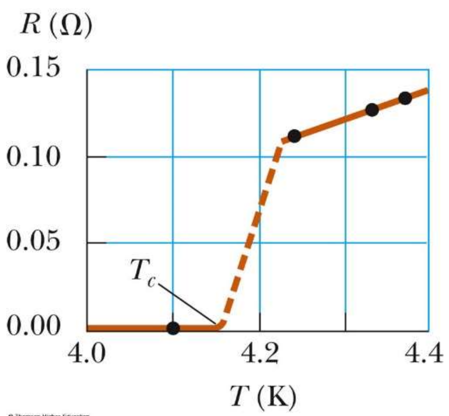
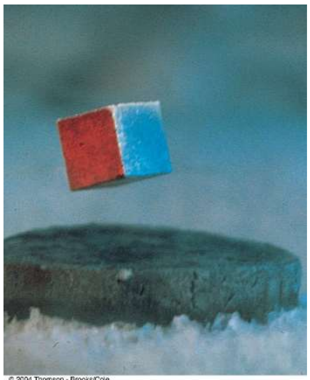


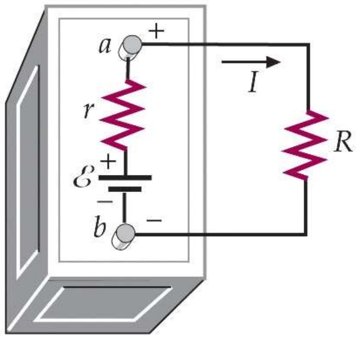
TABLE 27.3

Critical Temperatures for Various Superconductors

Material	T_c (K)
$HgBa_2Ca_2Cu_3O_8$	134
Tl—Ba—Ca—Cu—O	125
Bi—Sr—Ca—Cu—O	105
$YBa_2Cu_3O_7$	92
Nb_3Ge	23.2
Nb_3Sn	18.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88



27.6 Electrical Power



Joule Heating:

$$\Delta U = \Delta V \Delta Q \quad \text{-->} \quad P = \frac{\Delta U}{\Delta t} = \Delta V \frac{\Delta Q}{\Delta t} = \Delta V I$$

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

Example: An electric heater is constructed by applying a potential difference of 120 V across a Nicrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

$$I = \frac{V}{R}$$

$$P = I^2 R$$

Example: An immersion heater must increase the temperature of 1.50 kg of water from 10ºC to 50ºC in 10.0 min while operating at 110 V. What is the required resistance of the heater?

$$P = \frac{mc\Delta T \times 4.18(J/cal)}{\Delta t} = \frac{(\Delta V)^2}{R} \quad \rightarrow \quad R = 28.9\Omega$$