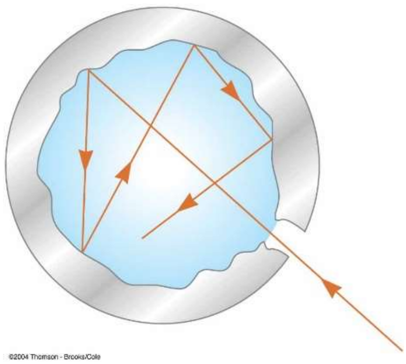
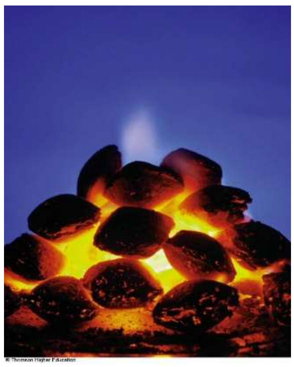
Chapter 40 Introduction to Quantum Physics

1900-1930 A new theory called quantum mechanics was highly successful in explaining the behavior of particles of microscopic size.

Because scientists learn the wave and particle natures of light in 19th century, they propose the same dual natures to particles. In addition, they believe that the wave nature shall be enhanced for particles of microscopic size just because of the coherence of their waves in such a small scale.

40.1 Blackbody Radiation and Plank's Hypothesis





1. Total power of the emitted radiation increase with temperature. Stem's Law: $P = \sigma A e T^4$
2. The peak of the wavelength distribution shifts to shorter wavelength as the temperature increases. Wien's displacement law: $\lambda_{\max} T = 2.898 \times 10^{-3} m \cdot K$

Classical Approach:

$S = k \ln \Omega(N, V, E)$ → $P(N, V, E) \propto e^{S/k}$ or $P(N, V, E) = Ce^{-\Delta S/k}$

From $dE = dQ + dW + \mu dN = TdS - PdV + \mu dN$, we have

$$\Delta S = \frac{1}{T}(\Delta E + P\Delta V - \mu \Delta N)$$

Canonical Ensemble:

$P(N,V,T) = Ce^{-\Delta E/kT}$ (or classical statistics, Boltzmann distribution)

The Boltzmann distribution:
The probability of finding a particle at energy E is

$$P(E) \propto \exp\left(-\frac{E}{kT}\right).$$

$R(1+2 \to 3+4) = Cn(E_1)n(E_2) \leftrightarrow R(3+4 \to 1+2) = C'n(E_3)n(E_4)$
principle of microscopic reversibility: $C = C'$ → $n(E_1)n(E_2) = n(E_3)n(E_4)$
→ $\ln(n(E_1)) + \ln(n(E_2)) = \ln(n(E_3)) + \ln(n(E_4))$
Elastic scattering: $E_1 + E_2 = E_3 + E_4$
The only way to satisfy the two eqs is $\ln(n(E)) = A - \beta E$ → $n(E) = A\exp(-\beta E)$

Normalize to get the constant A by $\int_0^\infty Ae^{-\frac{E}{kT}}dE = 1$ → $A = \frac{1}{kT}$

$P(E) = \frac{1}{kT}e^{\frac{-E}{kT}}$ use integration

$$\langle E \rangle = \int_0^\infty EP(E)dE \Big/ \int_0^\infty P(E)dE = -kT\frac{d}{d\alpha}\left(\ln\left(\int_0^\infty \frac{1}{kT}e^{-\alpha\frac{E}{kT}}dE\right)\right)_{\alpha=1} = kT$$

The probability of finding a particle with a speed between v and v+dv is

$$N_v = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^2 e^{-\frac{mv^2/2}{kT}}$$

Electromagnetic waves are confined in a 1D box with a length of L.
$E(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t) = 2A\sin(kx)\cos(\omega t)$ and $kL = n\pi$ →

$k = n\frac{\pi}{L}$ → $\Delta n = \frac{\Delta k}{(\pi/L)}$

For a 3D box: $\Delta N = \frac{4\pi k^2(\Delta k)}{(\pi/L)^3} \times \frac{1}{8} = \frac{\pi k^2(\Delta k)}{2(\pi/L)^3}$

Consider two possible polarizations, the number of states shall be:

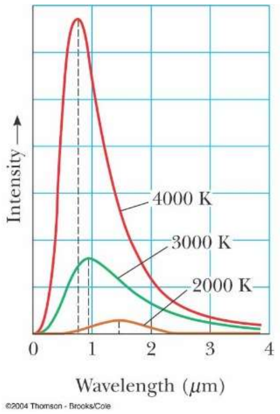
$$\rho(k,T)dk = \frac{\langle E \rangle}{V}dk = 2\times \left. \frac{\pi k^2 dk}{2(\pi/L)^3}\times k_B T \middle/ L^3 \right. = \frac{k^2 dk}{\pi^2}\times k_B T$$

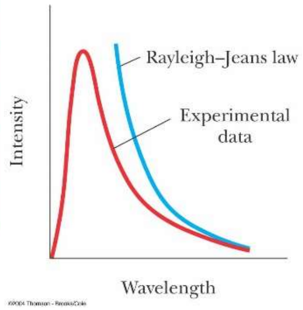
$$\rho(k,T)dk = \rho(\lambda,T)d\lambda = -\frac{8\pi}{\lambda^4}k_B T d\lambda$$

change the direction of the integration

$$I(\lambda,T) = \frac{c}{4}\rho(\lambda,T) = \frac{2\pi c k_B T}{\lambda^4}$$

(Rayleigh-Jeans Law, SI → Gaussian unit)





Quantum Approach (Max Plank):

1. The energy of an oscillator can have only certain discrete values $E_n = nhf$. n is a positive integer called **quantum number**. The energy is **quantized**. Each discrete value corresponds to a different **quantum state**.
2. The amount of energy emitted by the oscillator and carried by the quantum of radiation is $E = hf$.

Plank's const: $h = 6.626\times 10^{-34}$ J s

$$P(E) = \alpha e^{\frac{-nhf}{kT}} \quad \rightarrow \text{ use summation } \sum_{n=0}^{\infty}\left(e^{\frac{-hf}{kT}}\right)^n = \frac{1}{1-e^{-\frac{hf}{kT}}}$$

$$\langle E \rangle = \sum_{n=0}^{\infty}\left(nhfe^{-\frac{nhf}{kT}}\right) \Big/ \sum_{n=0}^{\infty}\left(e^{-\frac{nhf}{kT}}\right) = kT^2\frac{\partial}{\partial T}\ln\left(\sum_{n=0}^{\infty}\left(e^{-\frac{nhf}{kT}}\right)\right) = \frac{hf}{e^{hf/kT}-1}$$

$$\langle E \rangle = \frac{hf}{e^{hf/kT}-1} \quad kT >> hf \rightarrow \frac{hf}{e^{hf/kT}-1} = \frac{hf}{1+(hf/kT)-1} = kT \quad \text{(classical limit)}$$

$$\rho(\lambda,T)d\lambda = \frac{8\pi}{\lambda^4}\frac{hf}{e^{hf/kT}-1}d\lambda \quad \rightarrow \quad \rho(\lambda,T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

$$I(\lambda,T) = \frac{c}{4}\rho(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$$

Example:

(a) Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35ºC.

$\lambda_{\max}T = 2.898\times 10^{-3}\, m\cdot K$ → $\lambda_{\max} = 9.4\,\mu m$ → Infrared light

(b) Find the peak wavelength of the blackbody radiation emitted by the tungsten filament of a lightbulb which operates at 2000 K.

$\lambda_{\max} = 1.4\mu m$

(c) Find the peak wavelength of the blackbody radiation emitted by the sun (5800 K).

$\lambda_{\max} = 0.5\mu m$ → blue light

Example: The Quantized Oscillator

A 2.0 kg block is attached to a massless spring that has a force constant of k = 25 N/m. The spring is stretched 0.40 m from its equilibrium and released from rest.

(a) Find the total energy of the system and the frequency of oscillation according to classical calculations.

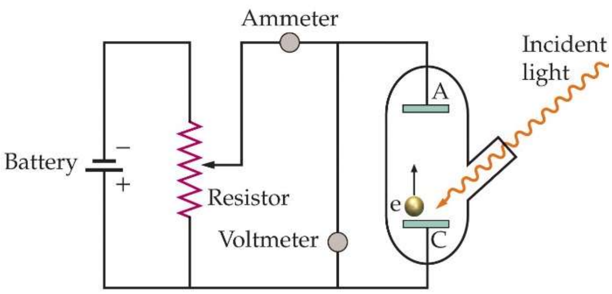
$$E = \frac{1}{2}kx^2 = \frac{1}{2}25(0.4)^2 = 2 \quad \text{J}$$

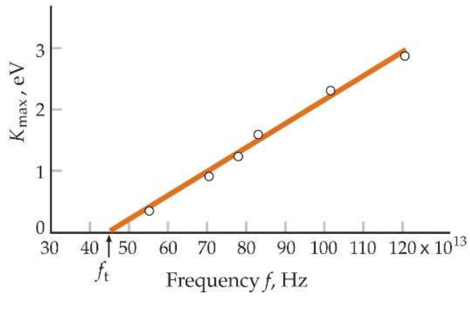
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{2}} = 0.56 \quad \text{Hz}$$

(b) Assume the energy of the oscillator is quantized, find the quantum number n for the system oscillating with this amplitude.

$$n = \frac{E_n}{hf} = \frac{2}{6.626\times10^{-34}\times0.56} = 5.4\times10^{33}$$

40.2 The Photoelectric Effect

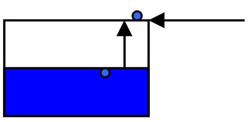




Energy of Photons:

$E = hf$, $h = 6.626\times10^{-34}$ J s

Required energy to move an electron out of the metal – Work function ϕ



$K_{\max} = \frac{1}{2}mv^2 = hf - \phi = e\Delta V_s$, ΔV_s is the stopping potential.

Required energy for an electron to be removed from the metal: $hf - \phi \geq 0$

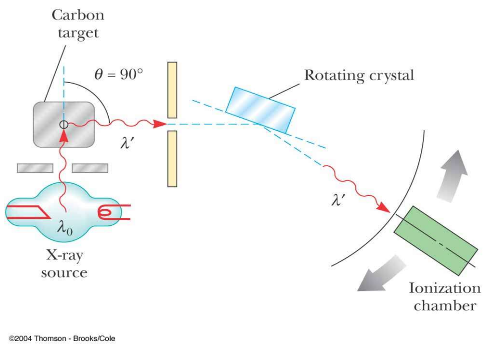
$$hf_t - \phi = 0 \quad \text{-->} \quad f_t = \frac{\phi}{h}$$

Example: Calculate the photon energies for light of wavelengths 400 nm (violet) and 700 nm (red). $E = hf = h\dfrac{c}{\lambda}$

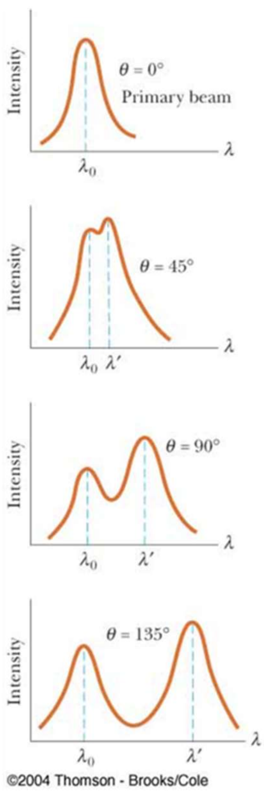
Example: The intensity of sunlight at the earth's surface is approximately 1400 W/m^2. Assume the average photon energy is 2 eV (corresponding to a wavelength of approximately 600 nm), calculate the number of photons that strike an area of 1 cm^2 each second.

$$\Delta E = 1400\frac{W}{m^2}\frac{1}{10000}m^2 \times 1(s)\,, \quad N = \Delta E \div \left(2 \times 1.602 \times 10^{-19}\right) = 4.37 \times 10^{17}$$

40.3 The Compton Effect

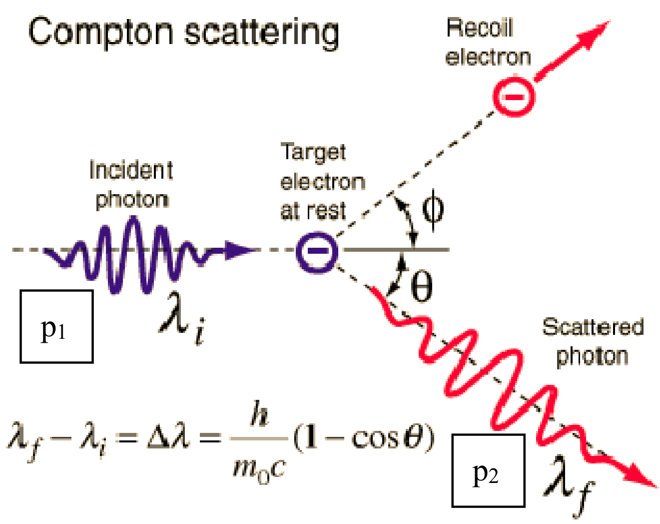


The scattering of photons from charged particles is called Compton scattering after Arthur Compton who was the first to measure photon-electron scattering in 1922. When the incoming photon gives part of its energy to the electron, then the scattered photon has lower energy and according to the Planck relationship has lower frequency and longer wavelength. The wavelength change in such scattering depends only upon the angle of scattering for a given target particle.

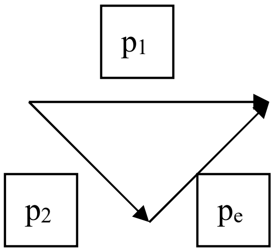


Why is the frequency of scattered light different from that of incident light?

The importance of $p = \dfrac{h}{\lambda}$



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$



For photons: $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

Momentum Conservation: $p_e^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta$

Energy of the scattered electron: $E = \sqrt{(m_0c^2)^2 + p^2c^2}$

Rest Energy: m_0c^2, Kinetic Energy: $K = pc$, Total Energy: $E = \sqrt{E_{rest}^2 + K^2}$

Energy Conservation: $E_{photon,1} + E_{electron} = E_{photon,2} + E_{electron}$

$p_1c + m_ec^2 = p_2c + \sqrt{(m_ec^2)^2 + (p_ec)^2}$ -->

$p_1^2c^2 + m_e^2c^4 + p_2^2c^2 + 2p_1m_ec^3 - 2p_2m_ec^3 - 2p_1p_2c^2 = m_e^2c^4 + c^2(p_e^2)$

$p_1^2c^2 + p_2^2c^2 + 2p_1m_ec^3 - 2p_2m_ec^3 - 2p_1p_2c^2 = c^2(p_1^2 + p_2^2 - 2p_1p_2\cos\theta)$

$2p_1m_ec - 2p_2m_ec - 2p_1p_2 = -2p_1p_2\cos\theta$

--> $\frac{1}{p_2} - \frac{1}{p_1} = \frac{1}{m_ec}(1-\cos\theta)$ --> $\lambda_2 - \lambda_1 = \frac{h}{m_ec}(1-\cos\theta)$

$\lambda_C \equiv \frac{h}{m_ec} = 2.43$ pm --> Can we use visible light to perform the experiments of Compton scattering?

$v \cong 0.9c \rightarrow \gamma = \frac{1}{\sqrt{1-(0.9)^2}} = 2.3,\ K = \gamma m_ec^2 - m_ec^2 \cong m_ec^2 \rightarrow hf = h\frac{c}{\lambda_e} = m_ec^2$

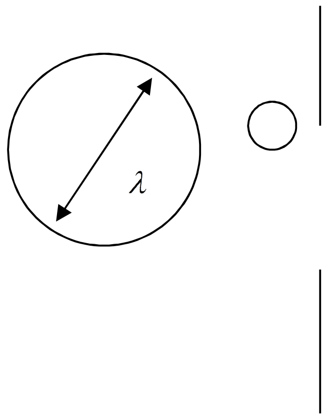
→ $\lambda_e = \frac{h}{m_e c}$

Example: The X-ray photon of wavelength 6 pm makes a head-on collision with an electron, so that the scattered photon goes in a direction opposite to that of the incident photon. The electron is initially at rest. (a) How much longer is the wavelength of the scattered photon than that of the incident photon? (b) What is the **kinetic energy** of the recoiling electron?

$$\lambda_2 - \lambda_1 = \lambda_C (1 - \cos(\pi)) = 2\lambda_C = 4.86 \text{ pm}$$

$$K_e = h\frac{c}{\lambda_1} - h\frac{c}{\lambda_2} = (1240 eVnm)\left(\frac{1}{0.006} - \frac{1}{0.01086}\right) = 92500 \text{ eV}$$

40.4 Photons and Electromagnetic Waves



The photoelectric effect and the Compton effect provide that the light behave as if it was a particle. The momentum of the light particle (photon) is P=E/c=h / λ.

Which model is correct? Is light a wave or a particle? The answer depends on the phenomena being observed.

We must accept both models and admit that the true nature of light is not describable in terms of any single classical picture.

The particle model and the wave model of light complement each other.

Particle Nature: E, p
Wave Nature: f, λ
Their Corresponding Relationship:

$$E = hf = \hbar\omega$$

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} = \frac{2\pi}{\lambda}\frac{h}{2\pi} = k\hbar$$

40.5 The Wave Properties of Particles

Because photons have both wave and particle characteristics, perhaps all forms of matter have both properties.

For a microscopic particle with E, P:

The wavelength of the photon wave: $\lambda = h / p$.

The frequency of the electron wave: $f = \dfrac{E}{h}$, the frequency of the particle is dependent on its energy

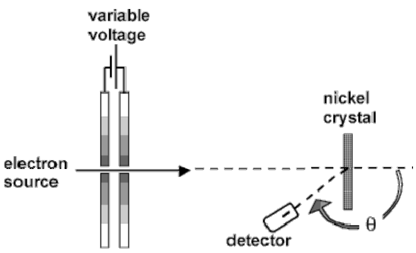
The particle can be described by the wave expression

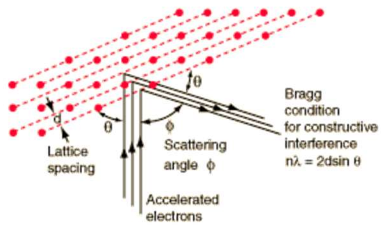
$$A\sin(kx - \omega t) = A\sin\left(2\pi\frac{p}{h}x - 2\pi\frac{E}{h}t\right).$$

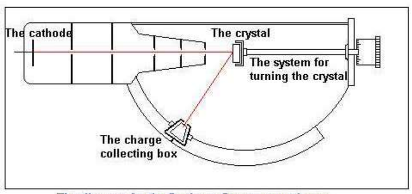
1923 – de Broglie's matter wave

1926 - Davisson and Germer succeeded in measuring the wavelength of electrons.

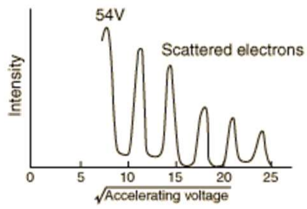
The Davisson-Germer Experiment







The diagram for the Davisson-Germer experiment.



Their results show conclusively the wave nature of electrons and confirmed the de Broglie relationship $p = h / \lambda$.

Example: Find the de Broglie wavelength of a 10^{-6} g particle moving with a speed of 10^{-6} m/s.

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{(10^{-9})(10^{-6})} = 6.626\times10^{-19} \text{ m}$$

Example: Calculate the de Broglie wavelength for an electron ($m_e = 9.11\times10^{-31}$ kg) moving at 1.00 X 10^7 m/s.

$$\lambda = h/p = h/(mv) = \frac{6.626\times10^{-34}}{9.11\times10^{-31}\times10^{7}} = 7.28\times10^{-11}\,\text{m}$$

For a low energy electron with kinetic energy K, its momentum is found from

$$K = \frac{p^2}{2m} \quad \text{-->} \quad p = \sqrt{2mK}$$

The wavelength is $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$. --> $\lambda = \frac{1.226}{\sqrt{K}}$ nm (K in electron volts)

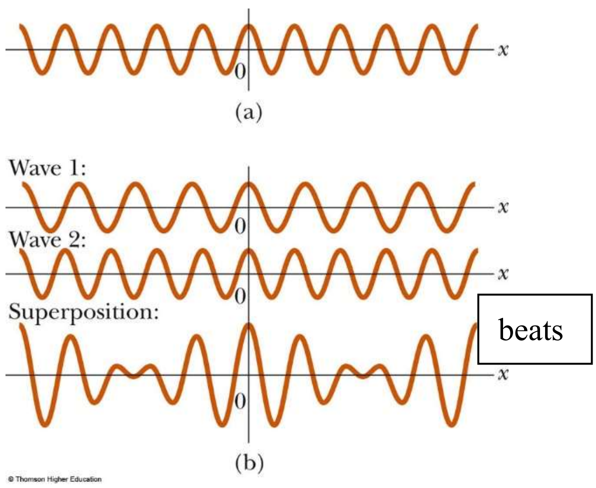
40.6 The Quantum Particle

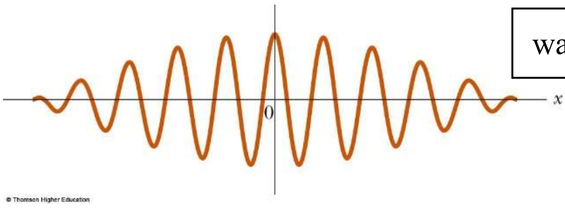
The quantum particle is a combination of the particle model (Chapter 2) and the wave model (Chapter 16).

ideal particle: localized in space
ideal wave: single frequency, infinitely long, delocalized in space

If a large number of waves are combined, the result is a wave packet which represents a particle.

From wave packet to continuous wave? → particle to wave
From continuous wave to wave packet? → wave to particle





wave packet

What are the wave velocity and particle velocity of the light?

wave velocity → phase velocity

particle velocity → group velocity

$$y_1 = A\cos(k_1 x - \omega_1 t),\quad y_2 = A\cos(k_2 x - \omega_2 t) \;\rightarrow$$

$$y = 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right)$$

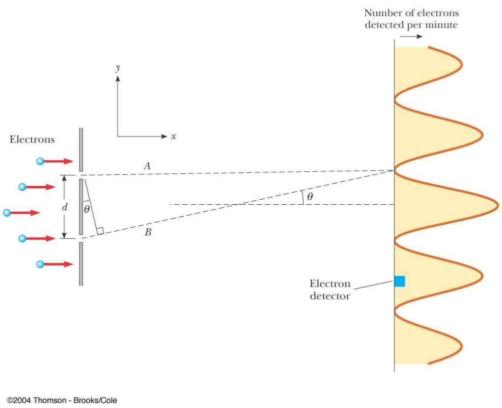
$$v_{phase} = \frac{\omega}{k}$$ - phase velocity

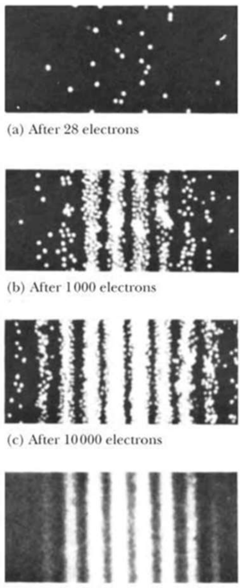
$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$ - group velocity – particle velocity

For light: $v_{group} = \dfrac{d\omega}{dk} = \dfrac{d(ck)}{d(k)} = c$

For quantum particle: $v_{group} = \dfrac{d\omega}{dk} = \dfrac{d(\hbar\omega)}{d(\hbar k)} = \dfrac{dE}{dp} = \dfrac{d\left(\dfrac{p^2}{2m}\right)}{dp} = \dfrac{p}{m} = u$

40.7 The Double-Slit Experiment Revisited





(a) After 28 electrons

(b) After 1 000 electrons

(c) After 10 000 electrons

(d) Two-slit electron pattern

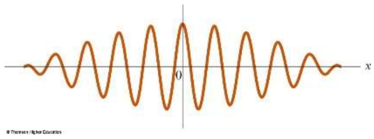
40.8 The Uncertainty Principle

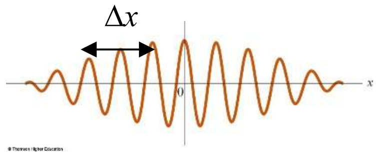
Extension of wave particle duality concept:

particle → localized in space

wave → delocalized in space

what about the determination of both wave and particle nature?





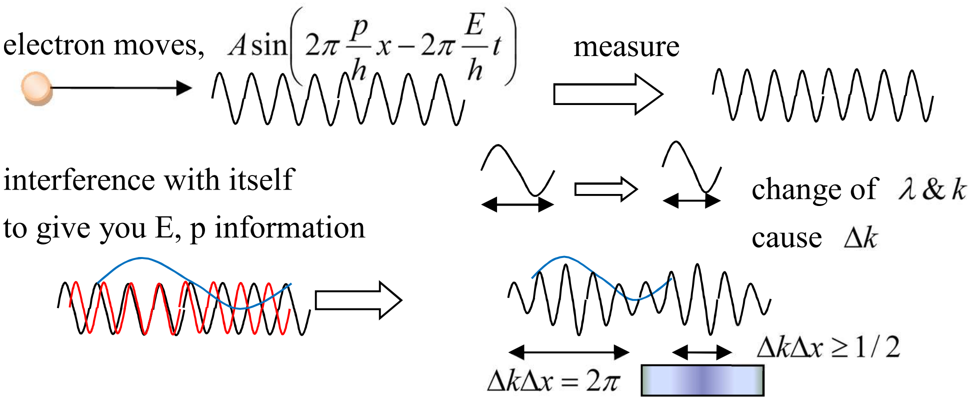
$\Delta k = 0 \rightarrow \Delta x = \infty$ wave

$\Delta k = \infty \rightarrow \Delta x = 0$ particle

$$A\sin(kx - \omega t) + A\sin((k + \Delta k)x - \omega t)$$

$$= 2A\sin\left(\frac{2k + \Delta k}{2}x - \omega t\right)\cos\left(\frac{\Delta k}{2}x\right) \approx 2A\cos\left(\frac{\Delta k}{2}x\right)\sin(kx - \omega t)$$

The Nature of interference with itself:



The uncertained position of this wave packet is Δx, $\Delta k\Delta x \geq \frac{1}{2}$

→ $\Delta p = (\Delta k)\hbar$ → $\frac{\Delta p}{\hbar}\Delta x \geq \frac{1}{2}$ → $\Delta p\Delta x \geq \frac{\hbar}{2}$

The spatial distribution of the photon: λ

The momentum carried by the photon is $p = \frac{E}{c} = \frac{hf}{\lambda f} = \frac{h}{\lambda}$

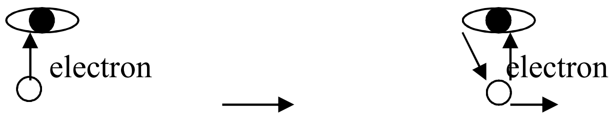
The product of the intrinsic uncertainties in position and momentum is:

$$\Delta x\Delta p \sim \lambda\frac{h}{\lambda} = h$$

The uncertainty principle: $\Delta x \Delta p \geq \frac{1}{2}\hbar$ → $\Delta E = \frac{p}{m}\Delta p$ → $\Delta x \frac{m}{p}\Delta E \geq \frac{1}{2}\hbar$

$\Delta x = v\Delta t$ → $v\Delta t \frac{m}{p}\Delta E \geq \frac{1}{2}\hbar$ → $\Delta E \Delta t \geq \frac{1}{2}\hbar$

Einstein's Picture:



Example: Locating an Electron

The speed of an electron is measured to be 5.00 X 10^3 m/s to an accuracy of 0.00300%. Find the minimum uncertainty in determining the position of this electron.

$$\Delta x \geq \frac{\hbar}{2} \frac{1}{9.11 \times 10^{-31} \times 5 \times 10^3 \times 0.00003} = 4 \times 10^{-4} \quad \text{m}$$