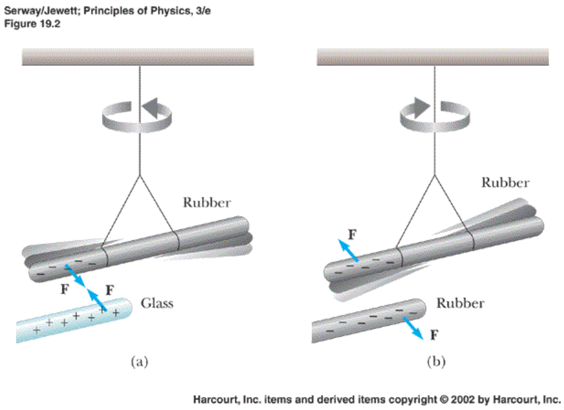
Chapter 22 Electric Fields

The electromagnetic force between charged particles is one of the fundamental force of nature.

22.1 Properties of Electric Charges

a rubber rod rubbed with fur → electrons are transferred from the fur to the rubber

a glass rod rubbed with silk → electrons are transferred from the glass to the silk



1. two kinds of charges: positive and negative
2. charges of the same sign repel one another
3. charges with opposite signs attract one another
4. electric charge is always conserved
5. electric charge is quantized

Rubbing Process

Electron Affinity:

$$X_g + e^- \to X_g^-$$

It is the energy released when the reaction happens.

$$F + e^- \xrightarrow{-328\_kJ/mol} F^-$$

Electrons are transferred from the material higher in the table to the lower rank material in the table.

Modeling the rubbing process:

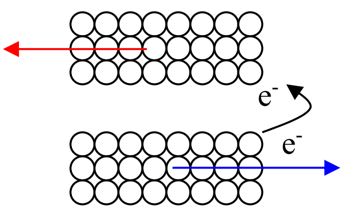
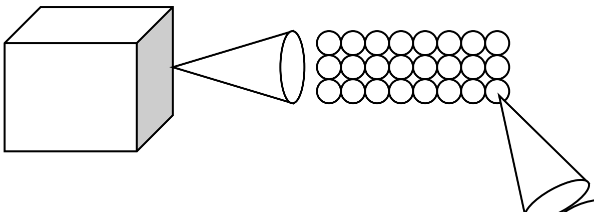


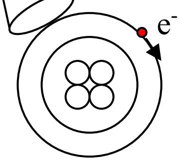
TABLE 21-1

The Triboelectric Series

+ Positive End of Series
Asbestos
Glass
Nylon
Wool
Lead
Silk
Aluminum
Paper
Cotton
Steel
Hard rubber
Nickel and copper
Brass and silver
Synthetic rubber
Orlon
Saran
Polyethylene
Teflon
Silicone rubber
– Negative End of Series

Charge Quantization





Atoms are electrically neutral.

Atom → Nucleus (protons and neutrons)

Protons are positively charged.

Charges of the proton and electron are $+e$ and $-e$, respectively.

e is called the fundamental unit of charge.

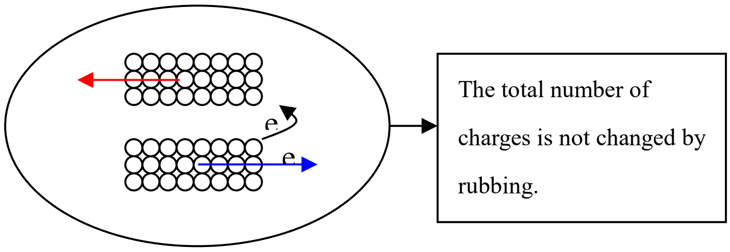
Any charge Q in nature can be written $Q = \pm Ne$, where N is an integer.

A rubber rod rubbed with fur transfers 10^{10} or more electrons to the rod.

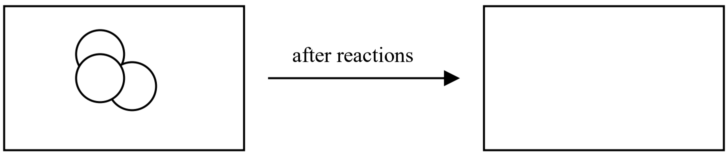
→ compared with the molar number of 10^{23} and the single atom number of 1

Charge Conservation

Charge is conserved.



Charges in the system are conserved.

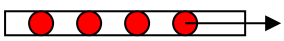


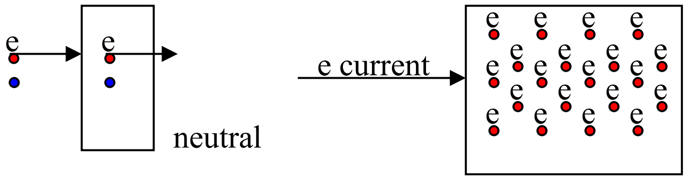
Example: $2H^{+} + O^{2-}$ → H_2O

Will the charge be conserved during nuclear reaction? →

law of conservation of charge

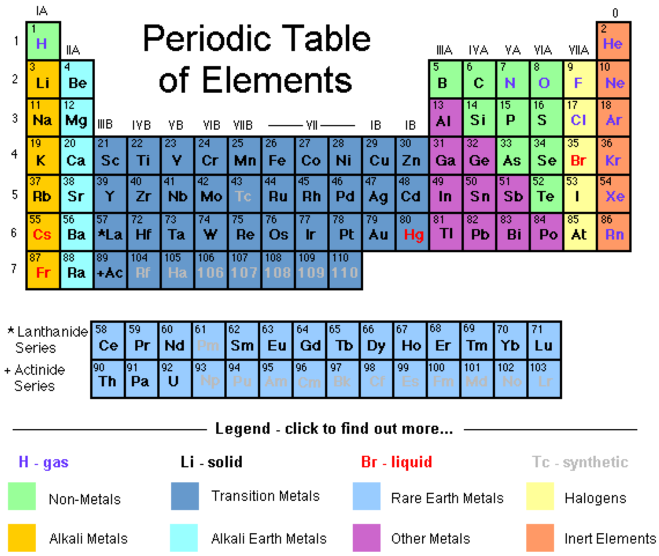
Current flow model:





The SI unit of charge:

$e = 1.602177 \times 10^{-19}$ C

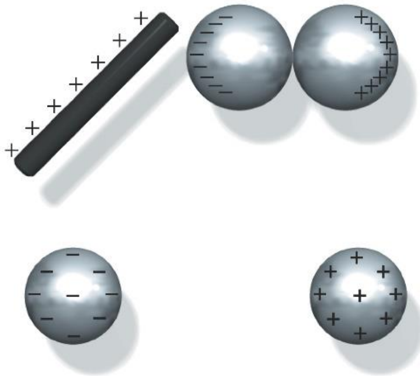


Exercise: A copper penny (Z = 29) has a mass of 3 grams. What is the total charge of all the electrons in the penny?

One copper atom has 29 protons and 29 electrons. (How do they distribute in the atom?)

$$Q = (-e)N_e = (-e)Z\frac{m}{M} \times N_A = -1.32\times10^5 \ \ \mathrm{C}$$

22.2 Charging Objects by Induction



Electrical conductors: (free electrons → to the boundary)

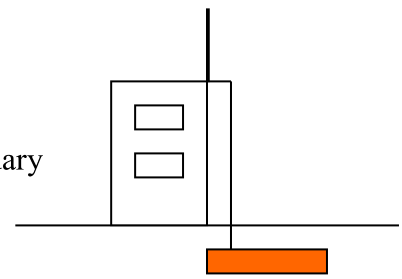
Having freely moving electrons.

Electrical insulators:

Electrons are bound to atoms and cannot move freely through the material.

Induction:

1. Electric force from the rod
2. Attract negative charges
3. Expel positive charges



When does a charge stop in the conductor? to the boundary

Grounded: If a conductor is connected to the earth, it is grounded.

What do you mean "connected to the earth"?

22.3 Coulomb's Law

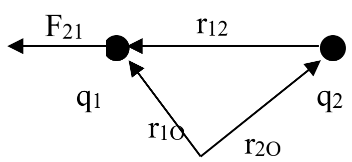
Charles Coulomb (1736 - 1806)
The force exerted by one point charge on another acts along the line between the charges. It varies inversely as the square of the distance separating the charges and is proportional to the product of the charges. The force is repulsive if the charges have the same sign.

$\vec{C} = \overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{BO} = \overrightarrow{BA}$ (draw an arrow from B (point) to A)

$\vec{C} = \vec{r}_{AO} - \vec{r}_{BO} = \vec{r}_{AO} + \vec{r}_{OB} = \vec{r}_{AB}$ (relative vector, draw an arrow from B (point) to A)

$\vec{F}_{21} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$, where $k = 8.99 \times 10^9$ N m^2/C^2

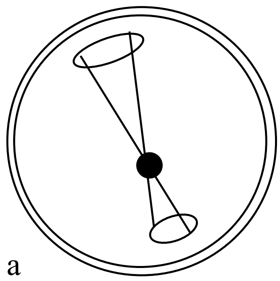
$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$



permittivity of free space: $\varepsilon_0 = 8.8542 \times 10^{-12} C^2 / N \cdot m^2$

for an electron, $q = -e = -1.602 \times 10^{-19}$ C

a force is a vector quantity
The Coulomb's law applies exactly only to particles.
(Notice: The vector direction of the displacement $\vec{r}_{12}$ is an arrow from 1 to 2. If you define a displacement vector as $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$, you may get the opposite answers.)



Do you believe the inverse r square law? Why not inverse r cubic law? How can we examine the law?

Example: The hydrogen atom
The electron and proton o a hydrogen atom are separated by a distance of approximately 0.53 angs. Find the magnitudes of the electrostatic force and the gravitational force that either particle exerts on the other.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(1.6 \cdot 10^{-19})^2}{(5.3 \cdot 10^{-11})^2} = 8.223 \cdot 10^{-8} N$$

Example: Compute the ratio of the electric force to the gravitational force exerted by a proton on an electron in a hydrogen atom.

$$\frac{F_e}{F_G} = \frac{\frac{ke^2}{r^2}}{\frac{Gm_p m_e}{r^2}} = \frac{ke^2}{Gm_p m_e} = \frac{(9\times10^9)(1.6\times10^{-19})^2}{(6.67\times10^{-11})(1.67\times10^{-27})(9.11\times10^{-31})} = 2.27\times10^{39}$$

Force Exerted by a System of Charges

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$

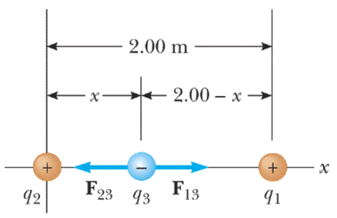
Resultant force on any one particle equals the vector sum of the individual forces due to all other particles. – principle of superposition

Example: Where is the resultant force zero?
Three charged particles lie along the x-axis. The particle with charge q1 = 15.0 μC is at x = 2.00 m, and the particle with charge q2 = 6.0 μC is at the origin. Where on the axis can a particle with negative charge q3 be placed that the resultant force on it is zero?

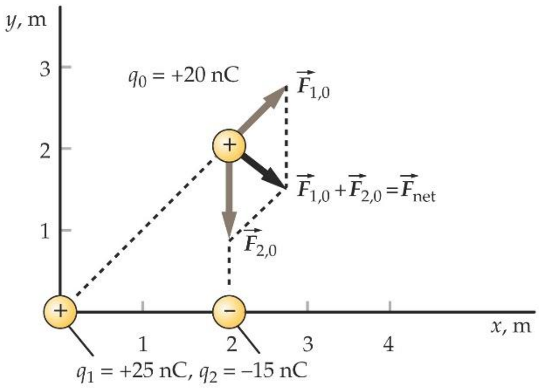
$$\frac{q_2 q_3}{x^2} = \frac{q_1 q_3}{(2-x)^2}, \quad (2-x)^2 \cdot 6 = x^2 \cdot 15$$

$$x = 0.775 \text{ m}$$

Serway/Jewett; Principles of Physics, 3/e
Figure 19.9



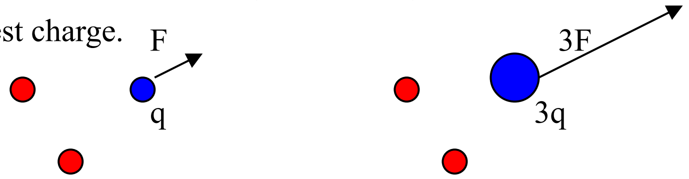
Example: Net Force in Two Dimension



22.4 The Electric Field

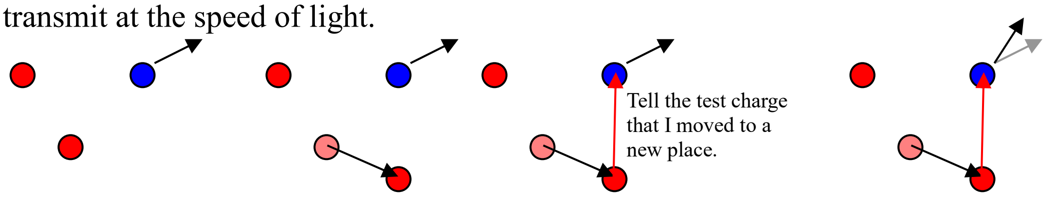
Coulomb' law for static electric charge → Action-at-a-distance law?
Why do we use the electric field instead of the electric force?

1. For a configuration of charges in the space, we can calculate the pulling force if we put the other charge (test charge) in the space. The force is proportional to the test charge.



2. Even though you do not put the test charge in the space, the mechanism to produce the force on the test charge still works in the space.

3. If you move the source charge to a new position, the force of charges at new positions will be felt by the test charge after a time of r/c. The field signal transmit at the speed of light.



The electric field $\vec{E}$ is defined to be $\vec{E} = \frac{\vec{F}}{q_0}$, where $\vec{F}$ is the net electric force and q_0 is the test charge.

TABLE 21-2

Some Electric Fields in Nature

	E, N/C
In household wires	10^{-2}
In radio waves	10^{-1}
In the atmosphere	10^{2}
In sunlight	10^{3}
Under a thundercloud	10^{4}
In a lightning bolt	10^{4}
In an X-ray tube	10^{6}
At the electron in a hydrogen atom	6×10^{11}
At the surface of a uranium nucleus	2×10^{21}

SI unit → N/C

In the beginning, we find the electric field $\vec{E}(\vec{r})$ in the space and we can then obtain the force exert on the charge q at any place.

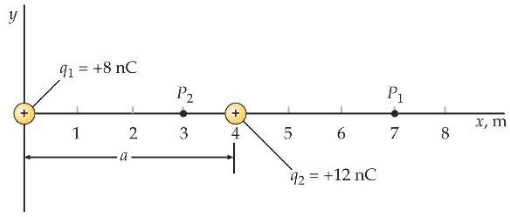
$$\vec{F}(\vec{r}) = q\vec{E}(\vec{r})$$

The Coulomb's force law for source charge q_s and test charge q_0 is $\vec{F}_{s0} = \frac{kq_s q_0}{r_{s0}^2}\hat{r}_{0s}$.

The Coulomb's field law: $\vec{E}_{s0} = \frac{kq_s}{r_{s0}^2}\hat{r}_{0s}$

The total electric field: $\vec{E} = \sum_i \frac{kq_i}{r_i^2}\hat{r}_{0i}$

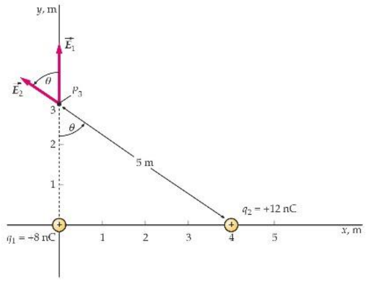
Example: A positive charge $q_1 = +8$ nC is at the origin, and a second positive charge $q_2 = +12$ nC is on the x axis at a = 4 m. Find the net electric field (a) at point P_1 and (b) at Point P_2.



$$\vec{E}_{P1} = \frac{(9\times10^9)(12\times10^{-9})}{(7-4)^2}\hat{i} + \frac{(9\times10^9)(8\times10^{-9})}{(7-0)^2}\hat{i} = 13.5 \quad \text{(N/C)}$$

$$\vec{E}_{P2} = \frac{(9\times10^9)(12\times10^{-9})}{(3-4)^2}(-\hat{i}) + \frac{(9\times10^9)(8\times10^{-9})}{(3-0)^2}\hat{i} = -100 \quad \text{(N/C)}$$

Exercise: Find the point on the x-axis where the electric field is zero.
Exclude the possibility at which $x > 4$ or $x < 0$.

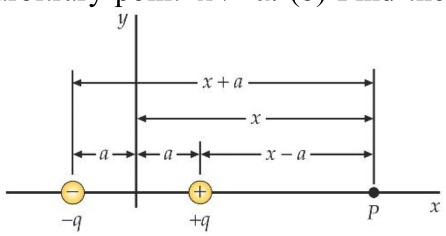


Example: Find the electric field on the y axis at y = 3 m.

$$\vec{E} = 0\hat{i} + \frac{(9\times10^9)(8\times10^{-9})}{(3)^2}\hat{j}$$

$$+\frac{(9\times10^9)(12\times10^{-9})}{(3^2+4^2)}\frac{4}{\sqrt{3^2+4^2}}(-\hat{i}) + \frac{(9\times10^9)(12\times10^{-9})}{(3^2+4^2)}\frac{3}{\sqrt{3^2+4^2}}(\hat{j})$$

Exercise: A charge $+q$ is placed at $x = a$ and a second charge $-q$ is placed at $x = -a$. (a) Find the electric field on the x axis at an arbitrary point $x > a$. (b) Find the limiting form of the electric field for $x \gg a$.



(a)

$$E(x) = \frac{kq}{(x-a)^2}\hat{i} + \frac{k(-q)}{(x-(-a))^2}\hat{i}$$

(b)

$$x >> a \;\rightarrow\; E(x) = \frac{kq}{(x-a)^2}\hat{i} + \frac{k(-q)}{(x-(-a))^2}\hat{i} \sim \left(\frac{kq}{x^2} - \frac{kq}{x^2}\right)\hat{i} = 0\,??$$

$$E(x) = \frac{kq}{(x-a)^2}\hat{i} + \frac{k(-q)}{(x-(-a))^2}\hat{i} = \frac{kq4ax}{(x^2-a^2)^2}\hat{i} \sim \frac{4kqa}{x^3}\hat{i}$$

22.5 Electric Field of a Continuous Charge Distribution

Electron charge is quantized with a fundamental unit of $e = 1.602\times10^{-19}$ C.
However, in the real world, we treat mm^3 or μm^3 as an infinitesimal volume, ΔV or dV.

Typically the electron concentration in metal is 10^{22} cm^{-3}, then we have $(10^{22})(10^{-4})^3 = 10^{10}$ electrons in the 1 μm^3-volume space. For this case, a change of one electron looks like continuous variation of charges. On the other hand, if we treat the 1 μm^3-volume space as small variable volume in comparison to the mm^3-volume space, we also can take it as a continuous distribution.

The charge per unit length is defined as $\lambda = \frac{\Delta Q}{\Delta l}$.

The surface charge density is $\sigma = \frac{\Delta Q}{\Delta a}$.

The volume charge density is $\rho = \frac{\Delta Q}{\Delta v}$.

Integration: if the charge Q is uniformly distributed on a line with a length of l →

$$\lambda = \frac{Q}{l},\quad Q = \int_0^l \frac{dQ}{dx}dx = \int_0^l \lambda dx = \lambda\int_0^l dx = \lambda l$$

If the total charge Q is uniformly distributed on a rectangle with an area of *a* X *b* →

$$\sigma = \frac{Q}{ab},\quad Q = \int_0^b\int_0^a \frac{dQ}{da}dxdy = \int_0^b\int_0^a \sigma dxdy = \sigma\int_0^b\int_0^a dxdy = \sigma ab$$

Calculating $\vec{E}$ From Coulomb's Law

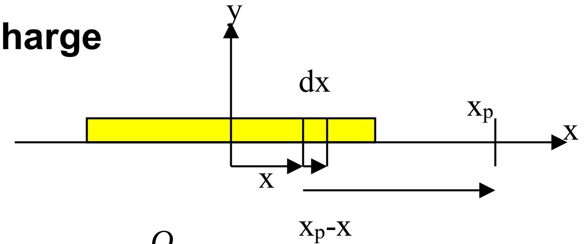
An element of charge $dq = \rho dV$ that is small enough to be considered a point charge

→ $d\vec{E} = \frac{kdq}{r^2}\hat{r}$

The total field at P is $\vec{E} = \int d\vec{E} = \int \frac{kdq}{r^2}\hat{r}$

$\vec{E}$ on the Axis of a Finite Line Charge

A charge Q is uniformly distributed along the x axis from $x = -L/2$ to $x = +L/2$.



The line charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. The charge of a small variable length dx can be estimated to be $\lambda dx = \frac{Q}{L} dx$

$$dE_x \hat{i} = \frac{kdq}{(x_p - x)^2}\hat{i} = \frac{k\lambda dx}{(x_p - x)^2}\hat{i}$$

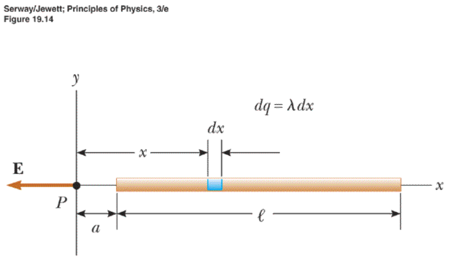
$$\vec{E} = \hat{i}\int dE_x = \hat{i}\int_{-L/2}^{L/2}\frac{k\lambda dx}{(x_p - x)^2} = \hat{i}\int_{-L/2}^{L/2}\frac{k\lambda dx}{(x - x_p)^2} = \hat{i}\int_{-L/2}^{L/2}\frac{k\lambda d(x - x_p)}{(x - x_p)^2} = \hat{i}k\lambda\left(\frac{(x - x_p)^{-1}}{-2+1}\right)_{x=-L/2}^{x=L/2}$$

$$= \hat{i}k\lambda\left(\frac{1}{x_p - L/2} - \frac{1}{L/2 + x_p}\right) = \hat{i}k\lambda\left(\frac{L}{x_p^2 - L^2/4}\right) = \hat{i}k\frac{Q}{x_p^2 - L^2/4}$$

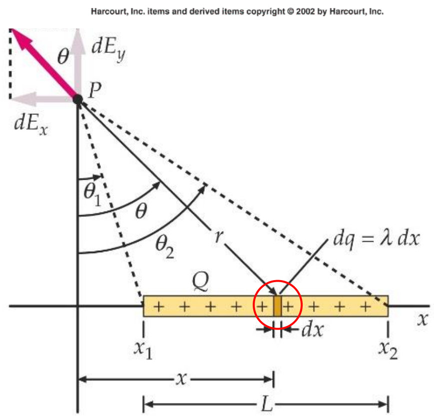
Example: The electric field due to a charged rod

$$\vec{E} = -\hat{i}\frac{1}{4\pi\varepsilon_0}\int_a^{a+l}\frac{\lambda}{x^2}dx = -\frac{\hat{i}\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{a+l}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 a(a+l)}\hat{i}$$



$\vec{E}$ off the Axis of a Finite Line Charge



$$d\vec{E} = \frac{k}{r^2}dq\hat{r} = \lambda\frac{k}{r^2}dx\hat{r}$$

$$d\vec{E} = \hat{i}dE_x + \hat{j}dE_y$$

$$dE_x = k\frac{\lambda dx}{(x^2+y^2)}\frac{x}{\sqrt{x^2+y^2}}$$

$$dE_y = k\frac{\lambda dx}{(x^2+y^2)}\frac{y}{\sqrt{x^2+y^2}}$$

$$\vec{E} = \int d\vec{E} = \int_{x=x1}^{x=x2}\left(-\hat{i}k\frac{\lambda dx}{(x^2+y^2)}\frac{x}{\sqrt{x^2+y^2}} + \hat{j}k\frac{\lambda dx}{(x^2+y^2)}\frac{y}{\sqrt{x^2+y^2}}\right)$$

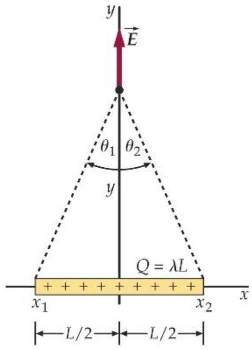
Find the reference side that do not vary with x → y

let $\frac{x}{y} = \tan\theta$

$$\vec{E} = \int d\vec{E} = \int_{\theta 1}^{\theta 2}\left(-\hat{i}k\frac{\lambda y\tan\theta y\sec^2\theta d\theta}{y^3\sec^3\theta} + \hat{j}k\frac{\lambda yy\sec^2\theta d\theta}{y^3\sec^3\theta}\right)$$

$$= \int_{\theta 1}^{\theta 2}\left(-\hat{i}k\frac{\lambda\sin\theta d\theta}{y} + \hat{j}k\frac{\lambda\cos\theta d\theta}{y}\right) = \frac{k\lambda}{y}\left(\hat{i}(\cos\theta_2 - \cos\theta_1) + \hat{j}(\sin\theta_2 - \sin\theta_1)\right)$$

Example: Electric Field on the Axis of a Finite Line Charge



From symmetry → $E_x = 0$

$$E_y = \int_{-L/2}^{L/2}\frac{k\lambda dx}{(x^2+y^2)}\frac{y}{\sqrt{x^2+y^2}}$$

$$x = y\tan\theta$$

$$E_y = 2k\lambda\int_0^{L/2}\frac{yy\sec^2\theta d\theta}{y^3\sec^3\theta} = \frac{2k\lambda}{y}(\sin\theta),\quad \sin\theta = \frac{L/2}{\sqrt{y^2+L^2/4}}$$

$$E_y = \frac{2k\lambda}{y}\frac{L/2}{\sqrt{y^2+L^2/4}}$$

$\vec{E}$ Due to an Infinite Line Charge

$$\vec{E} = \frac{k\lambda}{y}\left(\hat{i}(\cos\theta_2 - \cos\theta_1) + \hat{j}(\sin\theta_2 - \sin\theta_1)\right)$$

$$\theta_1 = -\pi/2,\quad \theta_2 = \pi/2$$

$$\vec{E} = \frac{k\lambda}{y}\left(0\hat{i} + 2\hat{j}\right)$$

$\vec{E}$ on the Axis of a Ring Charge

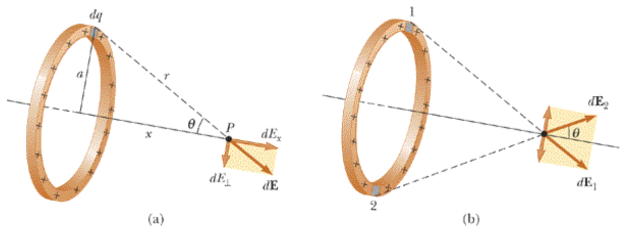
Example: The electric field of a uniform ring of charge

Serway/Jewett; Principles of Physics, 3/e
Figure 19.15

Charge density (charge per unit length) is $\lambda = \dfrac{Q}{2\pi a}$.

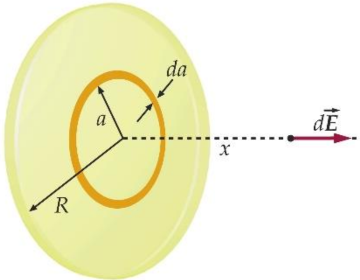
$$d\vec{E} = \frac{k}{r^2}dq\hat{r} = \frac{k\lambda}{r^2}ds\hat{r} = \frac{k\lambda}{r^2}ad\theta\hat{r}$$

$$\vec{E} = k\int_0^{2\pi}\frac{\lambda a d\theta}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}\hat{i}$$



$$\vec{E} = k\frac{xQ}{(x^2+a^2)^{3/2}}\hat{i}$$

$\vec{E}$ on the Axis of an Uniformly Charged Disk



Charge density (charge per unit area) is $\sigma = \dfrac{Q}{\pi R^2}$.

The total charge on a circle with the radius a is $dq = \sigma 2\pi a da$.

$$dE_x = k\frac{\sigma 2\pi a da}{(a^2+x^2)}\frac{x}{\sqrt{a^2+x^2}}$$

$$E = \int dE_x = \int_0^R k\frac{\sigma 2\pi a da}{(a^2+x^2)}\frac{x}{\sqrt{a^2+x^2}}$$

$$= 2\pi k\sigma x\int_0^R \frac{a da}{(a^2+x^2)^{3/2}} = 2\pi k\sigma x\left(\frac{1}{(a^2+x^2)^{1/2}}\right)_R^0 = 2\pi k\sigma x\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+R^2}}\right)$$

$\vec{E}$ Due to an Infinite Plane of Charge

$R >> x$ → $E \approx 2\pi k\sigma$ for x > 0

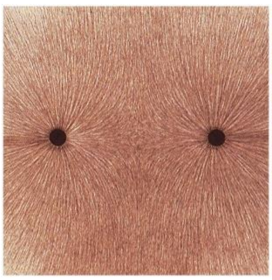
$E \approx -2\pi k\sigma$ for x < 0

22.6 Electric Field Lines

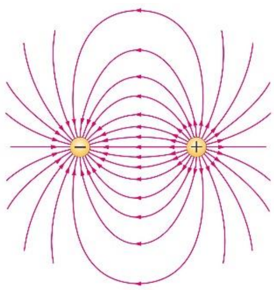
Electric field line → lines of force

Electric dipole? Monopole? → Sigle point charge?

Two positive point charges:



An electric dipole:

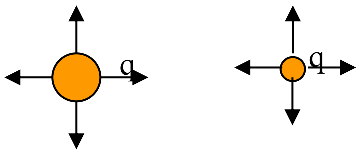


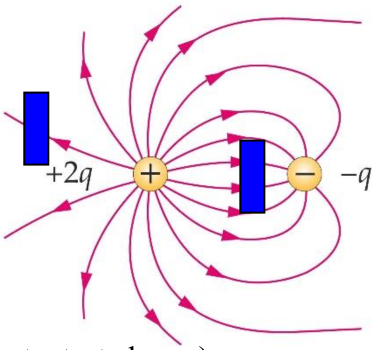
Rules for drawing electric field lines:

1. Electric field lines begin on positive charges (or at infinity) and end on negative charges.
2. The lines are drawn uniformly spaced entering or leaving an isolated point

charge.

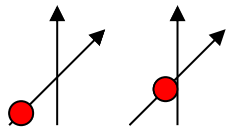
3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
4. The density of the lines at any point is proportional to the magnitude of the field at that point.
5. At large distances from a system of charges with a net charge, the field lines are equally spaced and radial, as if they came from a single point charge equal to the net charge of the system.
6. Field lines do not cross.



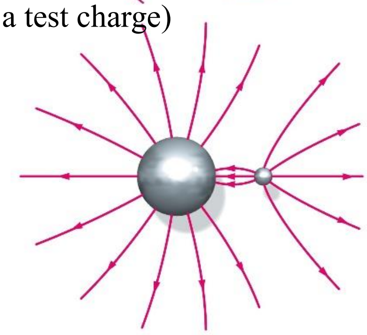


The strength or the magnitude of the field?

Why don't the field lines cross with each other? (just put a test charge)



Where should I go?



Example: What is the relative sign and magnitude

Electric fields in two-parallel and charged plates?

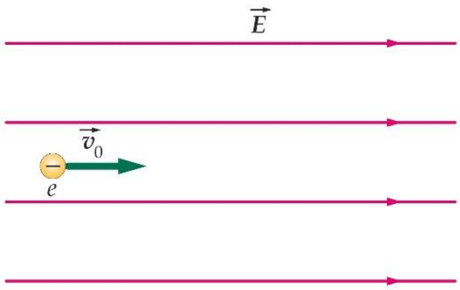
What is a uniform electric field?

22.7 Motion of a Charged Particle in a Uniform Electric Field

When a particle with a charge q is placed in an electric field $\vec{E}$, it experiences a force $q\vec{E}$. The particle has acceleration $\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}\vec{E}$.

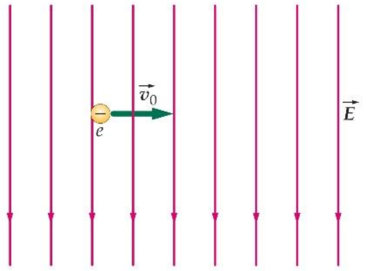
Example: An electron is projected into a uniform electric field $\vec{E} = 1000(N/C)\hat{i}$ with an initial velocity $\vec{v}_0 = 2\times10^6(m/s)\hat{i}$. How far does the electron travel before it is brought momentarily to rest?

$$a = \frac{-eE}{m_e},\quad 0 = v_0{}^2 + 2as \;\rightarrow\; s = \frac{v_0^2}{-2a} = \frac{v_0^2}{eE/m}$$

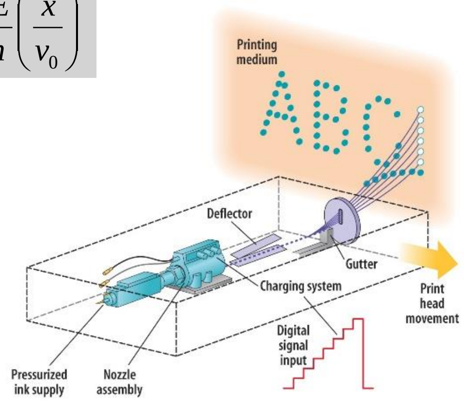


Example:

An electron enters a uniform electric field $\vec{E} = (-2000)(N/C)\hat{j}$ with an initial velocity $\vec{v}_0 = 10^6 (m/s)\hat{i}$ perpendicular to the field. By how much has the electron been deflected after it has traveled 1 cm in the x direction.



$$t = \frac{x}{v_0}, \quad \Delta y = \frac{1}{2}\frac{eE}{m}\left(\frac{x}{v_0}\right)^2$$

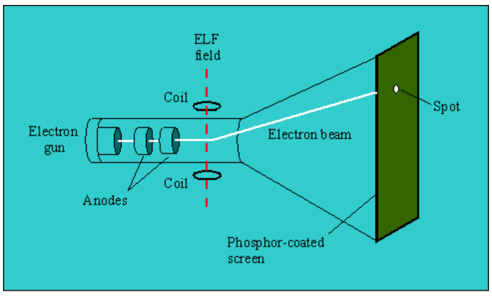


cathode ray tube

ref: http://whatis.techtarget.com/definition/0,,sid9_gci213839,00.html

A cathode ray tube (CRT) is a specialized vacuum tube in which images are produced when an electron beam strikes a phosphorescent surface. Most desktop computer displays make use of CRTs. The CRT in a computer display is similar to the "picture tube" in a television receiver.

A cathode ray tube consists of several basic components, as illustrated below. The electron gun generates a narrow beam of electrons. The anodes accelerate the electrons. Deflecting coils produce an [extremely low frequency] electromagnetic field that allows for constant adjustment of the direction of the electron beam. There are two sets of deflecting coils: horizontal and vertical. (In the illustration, only one set of coils is shown for simplicity.) The intensity of the beam can be varied. The electron beam produces a tiny, bright visible spot when it strikes the phosphor-coated screen.



To produce an image on the screen, complex signals are applied to the deflecting coils, and also to the apparatus that controls the intensity of the electron beam. This causes the spot to race across the screen from right to left, and from top to bottom, in a sequence of horizontal lines called the raster. As viewed from the front of the CRT, the spot moves in a pattern similar to the way your eyes move when you read a single-column page of text. But the scanning takes place at such a rapid rate that your eye sees a constant image over the entire screen.

The illustration shows only one electron gun. This is typical of a monochrome, or single-color, CRT. However, virtually all CRTs today render color images. These devices have three electron guns, one for the primary color red, one for the primary color green, and one for the primary color blue. The CRT thus produces three overlapping images: one in red (R), one in green (G), and one in blue (B). This is the so-called RGB color model.

In computer systems, there are several display modes, or sets of specifications according to which the CRT operates. The most common specification for CRT displays is known as SVGA (Super Video Graphics Array). Notebook computers typically use liquid crystal display. The technology for these displays is much different than that for CRTs.