Chapter 36 Interference of Light Waves

Physics II – Part III
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Conditions of Interference

The wave nature results in the interference and diffraction phenomena.

$$y_t(x,t) = y_1(x,t) + y_2(x,t)$$

$$\rightarrow \vec{E}_t(x,t) = E_1 \sin(kx - \omega t)\,\hat{\jmath} + E_2 \sin(kx - \omega t + \phi)\,\hat{\jmath}$$

The required conditions to observe the interference phenomena are:

1. The source light of interference must be coherent. The constant phases must be maintained in the light waves.
2. The light sources should be monochromatic, having a single wavelength.

The phase ϕ can be generated by either a time lag Δt or a path difference Δx.

$$\frac{\phi}{2\pi} = \frac{\Delta t}{T}, \frac{\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

Ref:

The Difference in Phase between Light Waves

Superposition and Interference

The planar light wave can be expressed as

$$\vec{E}(x,t) = E_0 \sin(kx - \omega t)\,\hat{\jmath} \;\&\; \vec{B}(x,t) = \frac{E_0}{c}\sin(kx - \omega t)\,\hat{k}$$

The interference of two light waves can be simply expressed by their electric field as

$$\vec{E}_t(x,t) = E_0 \sin(kx - \omega t)\,\hat{\jmath} + E_0 \sin(kx - \omega t + \phi)\,\hat{\jmath}$$

$$\vec{E}_t(x,t) = 2E_0 \cos(\phi/2)\sin(kx - \omega t + \phi/2)\,\hat{\jmath}$$

If the phase difference ϕ is zero, the superposition gives a constructive interference.

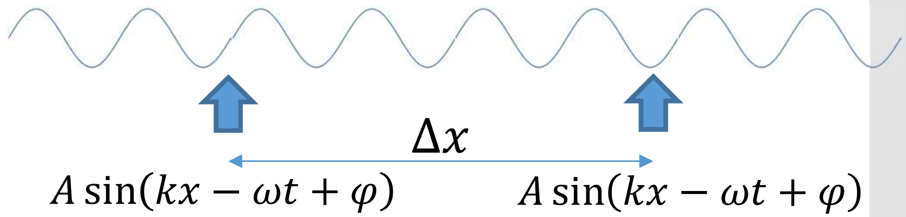
If the phase difference ϕ is π, the superposition gives a complete destructive interference.

Ref:

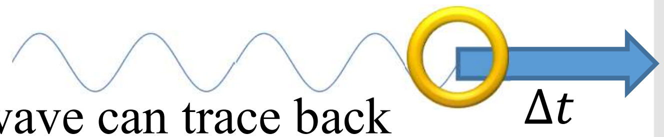
The Phase and Phase Difference

Phase Difference from Time Delay or Path Difference

The coherence of length Δx:
After traveling a distance Δx, the wave can trace back to its previous position and reach its previous condition.



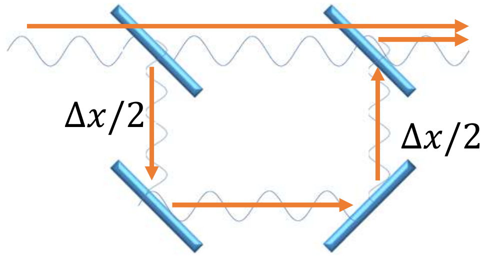
The coherence of time Δt:
After traveling a period of time Δt, the wave can trace back to its previous time and reach its previous condition.



The phase difference between two waves:

$$\psi_1(x,t)=A_1\sin(kx-\omega t)\,,\psi_2(x,t)=A_2\sin(kx-\omega t+\varphi)$$

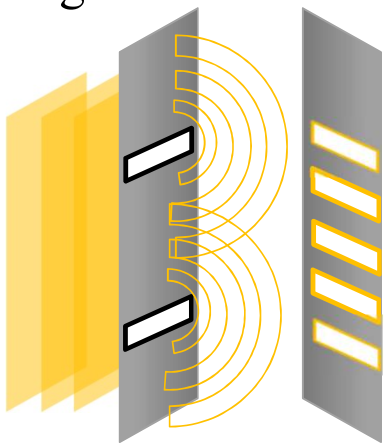
The phase difference could cones from a time delate $\varphi=\omega\Delta t$ or from a path difference $\varphi=k\Delta x$.

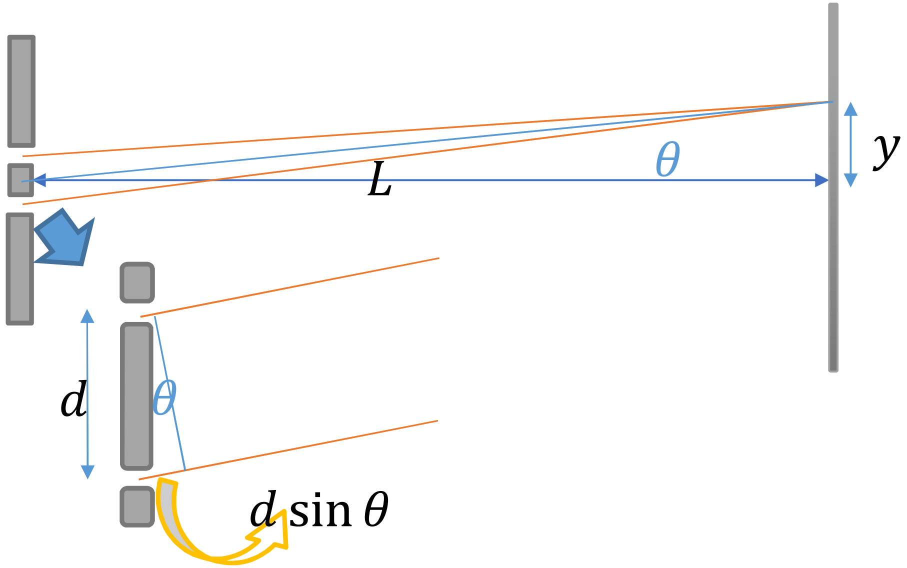


The Difference in Phase between Light Waves

Concepts of Double-Slit Interference

The interference due to two light waves:





The condition of $d \sin \theta = m\lambda$ gives a constructive interference.

The other condition of $d \sin \theta = (m + 1/2)\lambda$ gives a destructive interference.

$$\phi = 2\pi \frac{d \sin \theta}{\lambda}$$

$$\tan \theta = \frac{y}{L}$$

$$y_m = L \tan \theta_m \cong L \sin \theta_m = L \frac{m\lambda}{d}$$

Ref:

Intensity Distribution of The Double-Slit Interference Pattern

Double-Slit Interference

Assume the wave functions of light from the two slits are:

$$\psi_1 = A\sin(kx-\omega t)\,,\psi_2 = A\sin(kx-\omega t+\varphi)$$

Without interference, the total intensity of light from the two slits is

$$I_{total} = I_1 + I_2 = \varepsilon_0 C\langle A^2\sin^2(kx-\omega t)\rangle + \varepsilon_0 C\langle A^2\sin^2(kx-\omega t+\varphi)\rangle$$

$$I_{total} = \varepsilon_0 C\left(\frac{A^2}{2}+\frac{A^2}{2}\right) = \varepsilon_0 C A^2$$

Consider the interference effect, we have total wave function

$$\psi(x,t) = A\sin(kx-\omega t) + A\sin(kx-\omega t+\varphi)$$

$$\psi(x,t) = 2A\cos(\varphi/2)\sin(kx-\omega t+\varphi/2)$$

Double-Slit Interference

Intensity Distribution of The Double-Slit Interference Pattern

$$\psi(x,t) = 2A\cos(\varphi/2)\sin(kx - \omega t + \varphi/2)$$

$$I_{total} = \varepsilon_0 C\langle\psi^2(x,t)\rangle = 2\varepsilon_0 C A^2 \cos^2(\varphi/2)$$

$$\varphi = 2\pi\frac{d\sin\theta}{\lambda} \cong \frac{2\pi d}{\lambda}\tan\theta = \frac{2\pi d}{\lambda}\frac{y}{L}$$

$$I_{total} = 2\varepsilon_0 C A^2 \cos^2\left(\frac{\pi d y}{\lambda L}\right)$$

Note that the intensity at some place is two times of the light intensity without interference. The intensity at other places, however, is zero. The intensity averaged over the whole space gives $I_{total} = \varepsilon_0 C A^2$ which is the same as that of light without interference.

Phasor

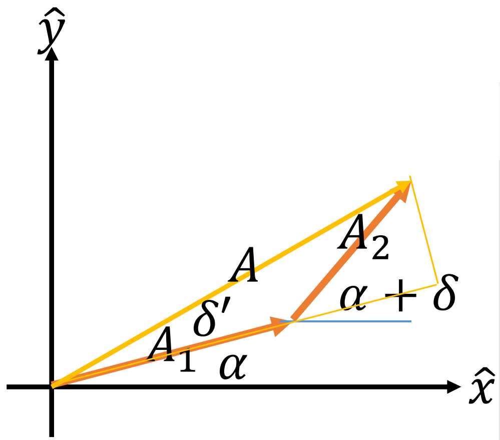
Phasor Used for Superposition of Light Waves

$$\psi_{total}(x,t) = \psi_1 + \psi_2$$

$$\psi_t = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \delta)$$

Let $kx - \omega t = \alpha$

$$\psi_t = A_1 \sin(\alpha) + A_2 \sin(\alpha + \delta) = A \sin(\alpha + \delta')$$



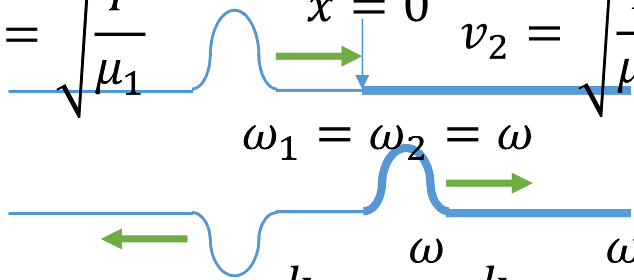
Draw the phasor plot like that shown on the right.

$$A^2 = A_1^2 + A_2^2 - 2A_1A_2\cos(\pi - \delta)$$

$$\tan\delta' = \frac{A_2 \sin\delta}{A_1 + A_2\cos\delta}$$

Change of Phase Due to Reflection

$$y_i(x,t) = A\cos(k_1 x - \omega t)$$
$$y_r(x,t) = B\cos(-k_1 x - \omega t)$$
$$y_t(x,t) = C\cos(k_2 x - \omega t)$$

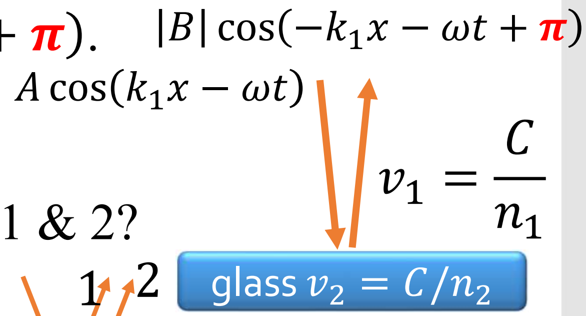


$$v_1 = \sqrt{\frac{T}{\mu_1}} \qquad x = 0 \qquad v_2 = \sqrt{\frac{T}{\mu_2}}$$
$$\omega_1 = \omega_2 = \omega$$
$$k_1 = \frac{\omega}{v_1}, k_2 = \frac{\omega}{v_2}$$

$$y_L = y_i + y_r = A\cos(k_1 x - \omega t) + B\cos(-k_1 x - \omega t)$$
$$y_R = y_t = C\cos(k_2 x - \omega t)$$
$$x = 0 \rightarrow A + B = C, k_1 A - k_1 B = k_2 C$$
$$C - B = A$$
$$k_2 C + k_1 B = k_1 A$$
$$B = \frac{k_1 - k_2}{k_1 + k_2} A, C = \frac{2k_1}{k_1 + k_2} A, if\ v_1 > v_2, k_1 < k_2\ \&\ B < 0$$
$$y_r(x,t) = -|B|\cos(-k_1 x - \omega t) = |B|\cos(-k_1 x - \omega t + \boldsymbol{\pi})$$

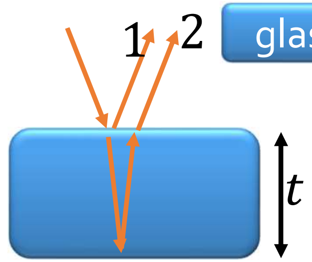
Interference in Thin Films

If $v_1 > v_2$, $y_r(x,t) = |B|\cos(-k_1 x - \omega t + \boldsymbol{\pi})$.



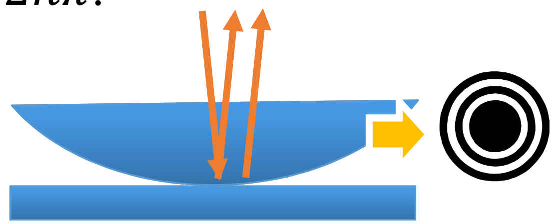
What is the phase difference between Light 1 & 2?

Light 1: π phase different from the incident wave
Light 2: $2\pi(2t/\lambda)$ different from the incident wave

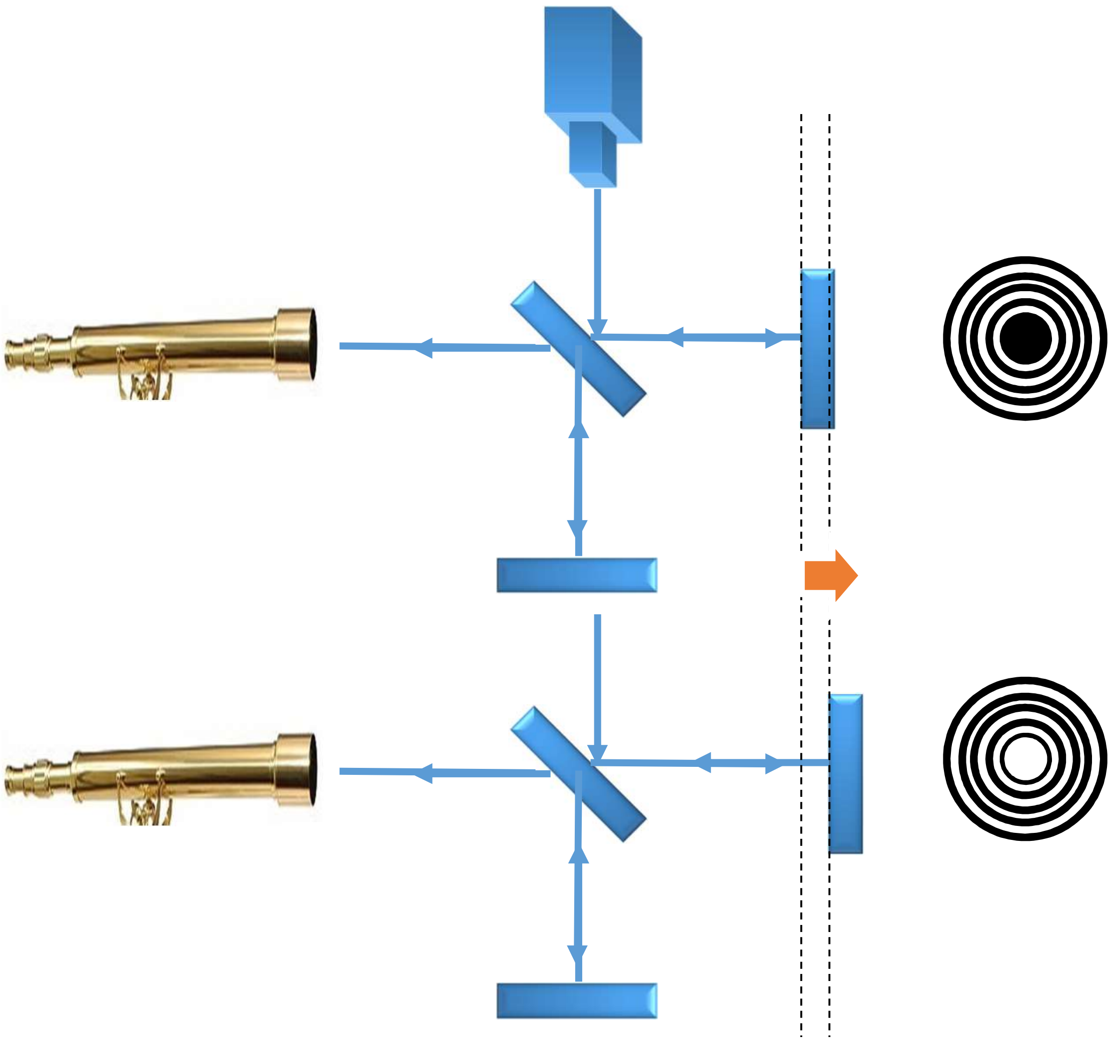


The two lights make a constructive interference when $\delta = 2\pi(2t/\lambda) = (2n+1)\pi$. It is destructive when $\delta = 2n\pi$.

The center region of the Newton ring is dark because the Light 1 reflected wave has a phase difference of π.



The Michelson Interferometer



Examples

What is the minimum path difference that will produce a phase difference of 180° for light of wavelength 800 nm? (b) What phase difference will that path difference produce in light of wavelength 700 nm?

(a) $\pi = 2\pi \frac{\Delta x}{\lambda} \rightarrow \Delta x = \frac{\lambda}{2} = 400$ nm

(b) $\varphi = 2\pi \frac{\Delta x}{\lambda} = 2\pi \frac{400}{700} = \frac{8\pi}{7}$

Ref:

Double-Slit Interference

Examples

Two narrow slits separated by 1.5 mm are illuminated by yellow light of wavelength 589 nm from a sodium lamp. Find the spacing of the bright fringes observed on a screen 3 m away.

The mth bright fringe occurs at $d \sin\theta = m\lambda$.

The angle θ also satisfies the condition of $\frac{y_m}{L} = \tan\theta$, where y_m is the distance between the mth fringe and the central fringe and L is the distance between the slit and the screen.

$$\sin\theta \cong \tan\theta \to \frac{m\lambda}{d} = \frac{y_m}{L} \to y_m = m\frac{L\lambda}{d} = m\frac{3\times589}{1.5\times10^6} = 0.0012m \text{ (m)}$$

$$y_1 = 0.0012 \text{ m}, y_2 = 0.0024 \text{ m}, \ldots$$

Ref:

Examples

Measuring The Wavelength of Laser Light

A laser is used to illuminate a double slit. The distance between the two slit is 0.03 mm. A viewing screen is separated from the double slits by 1.2 m. The second-order brighter fringe m = 2 is 5.1 cm from the center line. Determine the wavelength of the laser light.

$$d \sin\theta = m\lambda \ \& \ \frac{y_m}{L} = \tan\theta$$

$$\sin\theta \cong \tan\theta \rightarrow \frac{m\lambda}{d} = \frac{y_m}{L} \rightarrow \frac{2\lambda}{0.03 \times 10^6} = \frac{5.1 \times 10^{-2}}{1.2}$$

$$\lambda = 637.5 \text{ nm}$$

Ref:

Examples

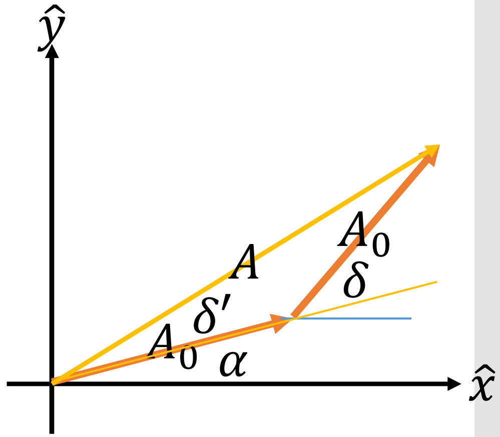
Use the phasor method to derive the superposition of two waves, $A_0 \sin(\alpha)$ and $A_0 \sin(\alpha + \delta)$, of the same amplitude.

$$\delta' = \delta/2$$

$$\frac{A}{2} = A_0 \cos(\delta') = A_0 \cos(\delta/2)$$

$$\psi_t = A_0 \sin(\alpha) + A_0 \sin(\alpha + \delta) = A \sin(\alpha + \delta')$$

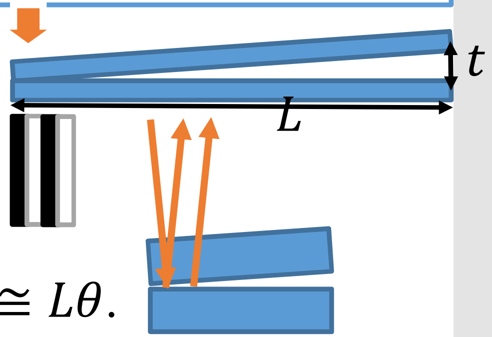
$$\psi_t = 2A_0 \cos(\delta/2) \sin(\alpha + \delta/2)$$



Ref:

Examples

A wedge-shaped film of air is made by placing a thin paper between the edges of two flat pieces of glass. Light of wavelength 500 nm is incident normally on the glass, and interference fringes are observed by reflection. If the angle θ made by the two plates is 3×10^{-4} rad, how many dark interference fringes per centimeter are observed?



The first interference strip is dark.

The first bright fringe occurs at $2t_1 = \lambda/2$.

The number of dark fringes is $N = 2t/\lambda$.

For a small angle θ, the separation distance t is $t \cong L\theta$.

The number of dark fringes per unit length is

$$\frac{N}{L} = \frac{2t}{\lambda L} = \frac{2L\theta}{\lambda L} = \frac{2\theta}{\lambda}, \lambda = 500 \text{ nm} = 5 \times 10^{-5} \text{ cm}$$

$$\frac{N}{L} = 2\frac{3 \times 10^{-4}}{5 \times 10^{-5}} = 12 \text{ cm}^{-1}$$